Multi-Plant Configuration
With Component Commonality

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An Analysis of Multi-Plant Configuration With Component Commonality

Abstract

We compare the benefits and costs of two alternative manufacturing network configurations in the presence of component commonality. We evaluate the trade-off between the decreased logistics costs and loss of risk-pooling benefits in plant networks which consolidate all component manufacturing within one plant (Product Network) as compared to those that do not (Process Network). We examine conditions when a Product Network may be chosen over a Process Network and vice-versa. We find that the risk pooling benefit provided by consolidating common subassembly production under one roof is reduced when the cost of acquiring common component capacity is sufficiently high *or* low. A post-optimality sensitivity analysis for the Process Network provides insight into subtle substitution effects, which are a direct outcome of cost mix differentials and network structure. Our results suggest that the impact of operational cost parameters on strategic decisions can often be non-intuitive. Overall, our analysis provides a link between strategic and operational decision-making in supply chain management, in the context of multi-plant configuration.

Key Words: Multi-Plant Networks, Configuration, Capacity Acquisition, Component Commonality, Supply Chain, Two-Stage Stochastic Programs.

1. Introduction

The evolution of multi-plant networks has been driven by financial, marketing and operational factors. For example, firms such as Toyota and BMW create a multi-plant network to leverage the exchange rate differentials across national boundaries. Others are motivated by production factors such as labor costs, logistics ease and efficiency, and market presence. Multiple plants generally offer benefits of increased responsiveness, ease in local customization and logistical gains afforded by simplification of the distribution network. However, accompanying these benefits is the increased managerial complexity associated with planning and operating multiple units.

In this paper we analyze the problem of deciding the configuration of multiple plants in a manufacturing network when products have component commonality. We compare two broad choices - product network configuration and a process network configuration. In a process network configuration, a firm chooses to make the common components in a "feeder" plant. This allows the firms to risk pool the uncertain demand for the component from all sources (products and markets). Hayes and Wheelwright (1984, p.91) suggest that the process network configuration would have less duplication of equipment for producing common parts. Indeed, the firm incurs the incremental logistics cost now associated with shipping components to the assembly plants. A firm may alternatively choose a product network configuration where all the plants within the network are assigned the responsibility of making the complete product under one roof.

Dieter (1997) presents the example of Seagate Inc., a multi-billion dollar hard disk drive (HDD) manufacturer and arguably the largest in the world, which can serve as a motivational example for this work. Seagate has set up a regional network of vertically integrated plants in Southeast Asia in order to reduce the supply disruptions that occur in the procurement of critical components from global supply chains. The two critical components of a HDD are the head and the recording media (DISC). Until now, "inductive thin film heads"(ITFH) were capable of sustaining increasing customer demands for more disk capacity. However, "magneto resistive heads" (MRH) manufactured using a new technology are now utilized in high capacity HDDs. Figure 1 shows the manufacturing supply chain structure of a hard drive manufacturer such as Seagate, based on the observations made by Dieter (1997). Plant 1 is dedicated to manufacturing recording media (DISC) -- a common component for any type of HDD. In the context of network configuration, this would amount to lower capacity investment for DISC production as the demand variance for both products is pooled. These components are then assembled into two different types of HDDs at Plant 2, an assembly plant. One wonders if this is the best way to organize manufacturing for the hard drive manufacturer.

INSERT FIGURE 1 HERE

This paper analyzes the tradeoffs incurred in making a choice on the configuration of multiple plants. How would a company decide on configuring its multi-plant network when there are two or three, key but disparate technologies/operations involved? How does a company decide on the configuration of its subassembly and assembly plants? What are the operational trade-offs? How do demand uncertainty and the presence of a common component affect the configuration choice? Are there specific advantages to risk pooling in the presence of demand uncertainty and component/subassembly commonality in the bill-of-materials structure?

2. Related Literature

The importance of the multi-plant configuration problem has been long recognized in operations strategy. Schmenner (1982) identifies four distinct multi-plant configuration strategies prevailing among Fortune 500 companies in the United States. Cohen and Lee (1989) extend Schmenner's work and formulate several plant charter strategies in global manufacturing networks:

- Regionalization Strategy, which focuses plant activity on the needs of a geographic region,
- Consolidation Strategy, which allows plants to manufacture for disparate markets with disparate processes under one roof,
- Product Focus Strategy, which establishes plants based on products or product lines,
- Process Focus Strategy, which establishes plants based on specific processes in the product value chain, and
- Vertical Integration Strategy, which assimilates vertically integrated processes under one roof.

It is clear that the network configuration problem for a manufacturing firm is both complex and critical. For example, the hard drive manufacturer in Figure 1 has several choices for configuring its plant network. It could follow a:

- Regionalization Strategy and setup components plants with production capability for DISC, MRH, and ITFH, and assembly plant s for MRH and ITFH assembly in each of the two markets,
- Consolidation Strategy and setup a single large plant with all capabilities for both markets,
- Product Focus Strategy and setup two plants, one with DISC, MRH, and MRH assembly capability and another with DISC, TFH and TFH assembly capability (each plant would serve both markets here),
- Process Focus Strategy, and setup 5 plants each specializing in DISC, MRH, TFH, MRH assembly and TFH assembly, or a
- Vertical Integration Strategy, and setup plants that are integrated by products (product focus strategy) or markets (regionalization strategy).

The various strategies outlined also serve as motivators for designing network configuration models.

Specific attempts have been made to resolve specific decisions such as which plant makes what and how much capacity do we acquire in each plant. For example, Moon (1989) and Cohen and Moon (1991) develop single period, deterministic mixed integer programming models for plant product mix loading and develop policies for optimal configuration of a set of capacitated plants. Cohen and Moon (1990) investigate the impact of scale economies, scope economies and supply chain transportation costs on network configuration and distribution policy with a mixed integer non-linear programming model and develop insights on the impact of changes in a firm's costs on its supply chain structure. Li and Tirupati (1995) and Li and Qui (1996) examine investment-planning problems, and determine the choice of technology and capacity additions to satisfy prescribed service levels in the presence of product families with dynamic demands. An insightful analysis of the multi-plant capacity choice problem can be found in Jordan and Graves (1995); they develop principles for configuring a pre-existing network of plants for near-optimal process flexibility at a fraction of the cost for total flexibility. Benjaafar and Gupta (1998) develop explicit expressions for the number of facilities (work centers) , the number of products assigned to each facility and their corresponding capacities, given a product mix for a plant.

Our work is similar to Harrison and Van Mieghem (1999) who develop a multi-period model for investing in resources within a single plant. They factor in demand uncertainty and derive qualitative insights into the invest-stay put-disinvest decision across multiple periods. They develop the structure of optimal policies and some approximate solutions. Recently, Van Mieghem and Rudi (2002) introduce a new class of models called “newsvendor networks”. These models can be utilized to study capacity investment under uncertainty whilst capturing features such a commonality, flexibility, substitution and transshipment. The models we present in this paper may be categorized as newsvendor networks. However we focus on investigating the impact of operational cost parameters (including logistics costs) on strategic configuration decisions in a multi-plant network. We consider commonality and logistics issues jointly and we show that besides inventory pooling commonality *also* results in subtle substitution effects, which go beyond pure risk pooling. To the best of our knowledge, the comparison of alternative network choices in the presence of commonality has not received adequate attention. Our models are stylized but offer some interesting insights, which would be very difficult to obtain with larger models.

Similar to Kulkarni, Magazine and Raturi (2001), we compare the choice of product- versus process-focused network for a single product, two-market scenario. They conclude that process-focused networks provide risk-pooling advantages when demand uncertainty is high and the logistics costs are low. They show that in spite of not considering economies of scale in their analysis, there are advantages to a process-focused configuration arising out of risk pooling over uncertain market demand. These advantages are dependent upon the configuration of the network. Also, when the penalty associated with market shortages are low, process-focused networks are preferred since the firm is more interested in acquiring capacity cautiously.

This paper extends their work in an important way. We compare the choice of process- versus product-focused network in the presence of component commonality. The specific scenario that we are concerned with here is that of a firm making two products with a common component, where the common component may be produced in the same plant as the final product (product focused network) or in a separate dedicated plant (process focused network). Overall, we assess the impact and importance of plant network configuration decisions in a firm’s supply chain management strategy.

The remainder of this paper is organized as follows. In section 3 we state our key assumptions and present the analytical models. In section 4 we present numerical illustrations and sensitivity analysis comparing the product configuration with the process configuration. In section 5 we conduct a sensitivity analysis for the optimal solutions to the process network configuration model as well as provide analytical results. Finally, in section 6, we offer a summary of our findings and suggest some directions for future research.

3. A Two-stage Model for the Plant Configuration problem

To limit our analysis, we compare the two alternatives mentioned in the earlier section – a process focused network and a product focused network. Figure 2 represents the structure of the former and Figure 3 the structure of the latter. We consider a firm that manufactures two products, both of which require two components. One of the components is common to both products (component C). The production process has two steps -- first, manufacture of the common component and then assembly of the two components (A and C, B and C) into the two products (AC and BC). Unique components A and B are acquired externally from vendors. The process-focused network has two plants, each of which focus on the two parts of the process – manufacturing (of C) and assembly (of AC and BC) as shown in Figure 2. The common component is shipped to the assembly plants (from Plant 2 to Plant 1) and finished products are shipped to the respective markets. The product focused network has two plants, each of which focuses on the end products (AC and BC) as shown in Figure 3. The common component C is manufactured in both plants in this case, and then assembled (into AC and BC) and shipped to the market.

INSERT FIGURES 2 AND 3 HERE

We define the following decision variables (see Figures 2 and 3 for clarification)

K_a = units of capacity acquired at Plant 1 for assembly of product AC,
K_b = units of capacity acquired at (Plant 1 for process focused network and Plant 2 for product focused network) for assembly of product BC,
K_c = units of capacity acquired at Plant 2 for manufacture of subassembly C in process focused network,
${}^{a}K_c$ = units of capacity acquired in Plant 1 for component C for assembly into product AC in a product focused network,
${}^{b}K_c$ = units of capacity acquired in Plant 2 for component C for assembly into product BC in a product focused network.

The only notational difference in the product focused network (Figure 3) is that we have superscripts representing particular end products for the common subassembly capacity i.e. ${}^{a}K_c$ and ${}^{b}K_c$. We separately define the amount of final product made as:

x_a = number of units of product "AC" assembled at Plant 1,
x_b = number of units of product "BC" assembled at Plant 1 for process focused network and Plant 2 for product focused network.

Note that x_a is the number of units of subassembly "C" manufactured in Plant 2 for shipping to Plant 1 and x_b is the number of units of subassembly "C" manufactured in Plant 2 for shipping to Plant 1 in Process focused network as shown in Figure 2. We specify $x_a \leq K_a$ and $x_b \leq K_b$ since it does not make sense to ship the common component in excess of the assembly capacity – indeed, shipping less than the assembly capacity is possible in many situations. Also, $(x_a + x_b) \leq K_c$ as the firm cannot produce more common component than the available capacity.

3.1 Objectives and Assumptions

We make two key assumptions. First, we model a two-product, two-plant, two-process scenario and assume a singular market (no demand substitution) for each product. We also assume that the manufacture of unique components (MRH and ITFH in the Seagate example) is outsourced. This allows us to focus on the benefits of dedicating a plant to producing the common component (process focus strategy) as compared to making it in assembly plants (product focus strategy).

Objectives: We formulate the decision-making problem associated with this network as a two-stage model. In Stage 1, the firm makes the capacity acquisition decision (K_a ,K_b , K_c) and in Stage 2 after the demands realize, the firm decides on it production plan (x_a , x_b), constrained by its earlier capacity acquisition decision. This hierarchical decision-making is found in a number of large firms including, Toyota, General Motors, and Seagate. The goal is to minimize the long-term network configuration and operational costs. The costs considered here consist of the cost of capacity acquisition, shipping, and penalty costs associated with shortages if demand for the end product (AC and BC) is not met.

Demand: There are two end products -- AC and BC. We assume independent random demands for the end products where the demand distributions are known and stationary. Further, there is no demand for components alone, for example, for service parts and replacements for the common component C. In case of the product focused network, the analytical results derived in this paper can be easily modified to incorporate correlated demands by using conditional density and distribution functions. Also, for the process focused network, the analytical results derived in this paper are valid even when demands are correlated. A discussion of the assumption of correlation of demand between AC and BC can be found in Kulkarni, Magazine and Raturi (2001) and Netessie, Dobson and Schumsky (2000).

Capacity: Investment in capacity acquisition is modeled using marginal costs (Cachon and Lariviere (1999), Fine and Freund (1990), Li and Tirupati (1995) and Van Mieghem (1998, 1999)). All capacity acquisition costs are convex (Fine and Freund (1990), Van Mieghem (1998)). We do not consider economies of scale in manufacturing C or assembling AC and BC as both of these would result in cost benefits accruing to the process plant networks. Thus the risk pooling benefits, that we believe accrue to the process plants network, would be further exaggerated for process plant networks.

Time: We develop single period stochastic models. By a single period, we do not mean the usual production period (say a month or a year), but rather a single life cycle of the product and/or technology. Since capacity acquisition is treated as a long-term decision, we believe that this assumption is not unreasonable. It is common to incorporate both capacity acquisition and production decisions into a single period two-stage model. (Van Mieghem (1998), Fine and Freund (1990)).

Costs: We do not consider holding costs, either average for the period, or end-of-period, as we have a single period model. There is no end of period inventory as production decisions are made after observing demand. Pipeline inventory costs as a result of shipping could be captured to some extent by appropriately inflating the relevant shipping costs parameters in the model. We assume that the labor content in manufacturing is low; hence there is no separate consideration of variable costs of manufacturing (Fine and Freund (1990)). We utilize a cost minimization approach with service levels being captured implicitly through penalty costs.

Parameters: We define the following parameters based on the assumptions above

s_c = cost of shipping one unit of subassembly "C" from Plant 2 to Plant 1.
s_{ac} = cost of shipping one unit of product "AC" from Plant 1 to market.
s_{bc} = cost of shipping one unit of product "BC" from Plant 1 or 2 to market.
p_{ac} = marginal penalty cost for shortages of product "AC".
p_{bc} = marginal penalty cost for shortages of product "BC".
C_i = marginal cost of capacity acquisition for capacity type i where,
    i = a (for assembly of product AC),
    i = b (for assembly of product BC),
    i = c (for manufacturing C))
D_{ac} , D_{bc} = random demands for products AC and BC respectively in the two markets.
$f(\cdot), g(\cdot)$ = Probability density functions characterizing demands for products AC and BC respectively.
$F(\cdot)$, $G(\cdot)$ = Distribution functions characterizing demands for products AC and BC respectively.

3.2 The Optimization problem for the Process Focused network

Based on our assumptions, the two-stage optimization problem associated with a Process focused network follows. In the following model, the Stage 1 objective function, denoted by W $(\cdot)$ represents the capacity acquisition costs and the expected operational costs. For the sake of simplicity, we assume linear capacity acquisition costs, although our analytical results are valid for any convex capacity acquisition cost function. The Stage 2 objective function (Π) represents the costs of shipping to the market and shortage costs.

Stage 1 :

$$
\begin{aligned}
& W(K^*) = Min.\ W(K_a, K_b, K_c) \\
& \qquad ST. \\
& \qquad\qquad K_a \le K_c,\ K_b \le K_c \\
& \qquad\qquad K_c \le K_a + K_b \\
& \qquad\qquad K_a, K_b, K_c \ge 0
\end{aligned}
$$

Stage 2 :

$$
\begin{aligned}
& \Pi = \ Min. \quad x_a(s_c + s_{ac} - p_{ac}) + x_b(s_c + s_{bc} - p_{bc}) + p_{ac}D_{ac} + p_{bc}D_{bc} \\
& \qquad ST. \\
& \qquad\qquad x_a \le Min(D_{ac}, K_a) \\
& \qquad\qquad x_b \le Min(D_{bc}, K_b) \\
& \qquad\qquad x_a + x_b \le K_c
\end{aligned}
$$

where $W(K_a, K_b, K_c) = E\ \Pi + (C_a K_a + C_b K_b + C_c K_c)$ and E represents the expectation operator.

If $\alpha = s_c + s_{ac} - p_{ac}$ and $\theta = s_c + s_{bc} - p_{bc}$ then in order to avoid trivial solutions we assume that $\alpha, \theta < 0$. We note that $-\alpha$ and $-\theta$ may be regarded as incremental revenues from the sub-networks pertaining to each market, if we view penalty costs as lost profits. Also $C_a + C_c + \alpha < 0$ and $C_b + C_c + \theta < 0$ since a violation of these assumptions would result in null capacity acquisitions. If the capacity costs in Stage 1 exceed, or are equal to the (shortage penalty cost - shipping cost) differential in Stage 2, then there is no incentive to acquire capacity since any shortage cost savings in the Stage 2 problem are forfeited in the capacity acquisition of the Stage 1 problem. The solution to the Stage 2 problem is obtained by simple inspection of the Stage 2 linear program. The solution is represented by the three cases below:

CASE 1 ($\alpha / \theta > 1$) $\quad x_a^* = Min(K_a, D_{ac}), \quad x_b^* = Min(K_b, D_{bc}, (K_c - x_a^*))$

CASE 2 ($\alpha / \theta < 1$) $\quad x_a^* = Min(K_a, D_{ac}, (K_c - x_b^*)), \quad x_b^* = Min(K_b, D_{bc})$

CASE 3 ($\alpha / \theta = 1$) No unique optimum but the form of the optimal solution to stage 2 is the same as that seen in the two earlier cases.

It suffices to consider just CASE 1 (i.e. $\alpha / \theta > 1$) to gain some insight into the optimal policies for the network configuration problem. The two stage stochastic convex program can be solved parametrically in terms of the capacity acquisitions K_a, K_b, and K_c. As suggested in Fine and Freund (1990), Van Mieghem (1998) and Harrison and Van Mieghem(1999), we can partition the demand space for the final products into convex polygonal domains inside which the shadow price for the Stage 2 problem are constant (see Figure 4). Further, based upon our earlier assumptions about the cost parameters, the capacity acquisition vector will have non-zero elements. An optimal capacity acquisition policy exists and is given by the solution to the equations[1]**:**

$$\Lambda\ P'_{\Omega_i} = \begin{pmatrix} -C_a + n_4 - n_6 \\ -C_b + n_5 - n_6 \\ -C_c + n_6 - n_5 - n_4 \end{pmatrix} and \begin{pmatrix} n_1(K_c - K_a) \\ n_2(K_c - K_b) \\ n_3(K_c - K_b - K_a) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where, P_{Ω_i} = vector of probability measure of demand domains, Λ = vector of shadow prices in the demand domains, and n_i = Lagrange multipliers for capacity constraints.

In Figure 4, the demand domains are characterized as follows.

$\Omega_0 = \{D_{ac} \le K_a\ , D_{bc} \le K_b, D_{ac} + D_{bc} \le K_c\ ; D_i\ \varepsilon\ R_+^{\ 2}\}$

$\Omega_1 = \{D_{ac} \le K_c - K_b\ , D_{bc} > K_b; D_i\ \varepsilon\ R_+^{\ 2}\}$

$\Omega_2 = \{D_{ac} + D_{bc} > K_c\ , D_{ac} \le K_a\ , D_{ac} > K_c - K_b\ ; D_i\ \varepsilon\ R_+^{\ 2}\}$

$\Omega_3 = \{D_{ac} > K_a\ , D_{bc} > K_c - K_a\ ; D_i\ \varepsilon\ R_+^{\ 2}\}$

$\Omega_4 = \{D_{ac} > K_a\ , D_{bc} \le K_c - K_a\ ; D_i\ \varepsilon\ R_+^{\ 2}\}$

INSERT FIGURE 4 HERE

Solution to the optimality equations results in the specific values of capacity acquired. Based upon the values of the input parameters and depending upon the values of the Stage 1 Lagrange multipliers, a number of cases can arise. Thus, the unique optimal capacity acquisition for the process focused network optimization model can take one of the following five forms:

a. $K_c^* = K_a^* + K_b^*$, in which case,

$C_c > -q\ \ P_{\Omega_3}, C_b < -q\ \ P_{\Omega_1}\ \&\ C_a < (q - a)P_{\Omega_3} - aP_{\Omega_4}$ are necessary conditions.

b. $K_c^* = K_a^*$, in which case,

$C_c < -q\ \ (P_{\Omega_2} + P_{\Omega_3}) \quad \& \quad C_a > (q - a)P_{\Omega_3}$ are necessary conditions.

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[1] Proofs of all the propositions and results in this paper can be obtained from the authors upon request.

c. $K_c^* = K_b^*$, in which case,

$C_c < -q\ (P_{\Omega_2} + P_{\Omega_3})$ is a necessary condition.

d. $K_c^* = K_a^* = K_b^*$ in which case,

$C_c < -q\ (P_{\Omega_2} + P_{\Omega_3})$ & $C_a > (q - a)P_{\Omega_3}$ are necessary conditions.

e. $K_c^* < K_a^* + K_b^*,\ K_c^* > K_a^*,\ K_c^* > K_b^*$.

Here P_{Ω_i} = vector of probability measure of demand domains. The above result highlights the important role of the relative values of marginal capacity acquisition costs. For example, consider the situation

$$-q\ P_{\Omega_3} < C_c < -q\ (P_{\Omega_2} + P_{\Omega_3}) \text{ and } (q - a)P_{\Omega_3} < C_a < (q - a)P_{\Omega_3} - aP_{\Omega_4}.$$

Then, depending upon the value of C_b, it can be shown that either {a}, {b} or {d} could be the optimal solution. However, the *optimal solution is unique and will take one of the forms given above.* The empirical intuition is as follows. Cases (b) and (c) are straightforward – the firm intends to serve a higher fractile of the demand for products AC and BC. In case (a), we acquire sufficient common component capacity to match the assembly capacity acquired for products AC and BC respectively. This represents the “high service” capacity scenario and represents the normal case in most firms. There is no benefit of risk pooling accruing in this scenario and component capacity is never the binding constraint. Case (d) is the “minimal service” scenario – we either provide service for one of the two products or marginal service for both. Case (e) represents the most interesting case -- component capacity is acquired based on the premise that risk pooling would reduce it from levels observed in case (a).

3.3 The Optimization problem for the Product Focused Network

The network configuration problem for the scenario in Figure 3 is also formulated as a two-stage model and is presented below.

Stage 1 :

$$
\begin{aligned}
& W(K_i^*) = Min.\ W_i(K_i, {}^iK_c) \\
& \quad ST. \\
& \qquad {}^iK_c \geq K_i \\
& \qquad K_i \geq 0\,,\ {}^iK_c \geq 0
\end{aligned}
$$

Stage 2 :

$$
\begin{aligned}
& \Pi_i = Min.\ x_i(s_{ic} - p_{ic}) + p_{ic}D_{ic} \\
& \quad ST. \\
& \qquad x_i \leq Min(D_{ic}, K_i)
\end{aligned}
$$

where, $W_i(K_i, {}^iK_c) = E\ \Pi_i + (C_iK_i + C_c\, {}^iK_c)$ and $i = a,b$

In the product focused network, $K_i = {}^iK_c$ at optimality. This is because any excess capacity over and above what is needed for assembly of AC and BC will not be utilized and will result in a sub-optimal solution. The optimal solution and the unique optimal capacity acquisition for the product focused network is given by,

$$\Pr(D_{ic} > K_i^*) = \frac{(C_c + C_i)}{(p_{ic} - s_{ic})}$$

where, i = a, b. This is a classic newsvendor type solution, wherein the critical capacity investment fractile balances overage and underage costs. Note that a product focused network with flexible capacity capable of manufacturing C and assembling both AC and BC can be easily considered within our modeling domain. We omit its analysis as it would digress from the focus of this paper, but details can be obtained from the authors upon request.

In order to understand the impact of the optimal capacity acquisitions on the overall network configuration decision, we conduct a numerical sensitivity analysis in the next section. We compare the results from both formulations of the facilities planning problem – process-focused network as well as the product-focused network. Also, in order to benchmark the solutions from the above formulations, we present numerical results from the case in which single stage models are considered. The analysis of such models is straightforward and is not discussed. As there are no second stage production decisions, the capacity acquired is also assumed to be the average quantity shipped. Hence, the common subassembly capacity acquired is equal to the sum of the two dedicated capacities. Thus, the numerical illustration in the next section gives useful insights into (1) the effectiveness of the two stage models, and (2) the conditions under which firms should consider process-focused networks over product-focused networks.

4. Numerical Illustration and Sensitivity

This section is important for two reasons. First, it allows us to gain insight into the relative levels of dedicated and common component capacity acquisition for the two different types of networks. Second, we can compare the Expected Total Cost (ETC) for the two network configurations at optimality and study its behavior with respect to important cost parameters. In effect, this allows us to derive qualitative insight into conditions under which a process focused network may be chosen over a product focused network and vice-versa.

Initially, we provide a simple example to illustrate the difference in the optimal capacity acquisitions of the two networks with alternative solution approaches. We use the following parameter values;

Costs of capacity per unit: $C_a = 5$, $C_b = 3$, $C_c = 2.5$,
Shortage costs per unit: $p_{ac} = 25$ and $p_{bc} = 15$.
Parameters for controlling shipping costs: $\alpha = -20$, $\theta = -10$,
Demand distribution: $D_{ac} \sim U(0, 3000)$,$D_{bc} \sim U(0, 1000)$

Table 1 compares the results for the product and process focused networks. First, for the single stage optimization for a process focused network, we acquire much less of all types of capacities as compared to two stage optimization of the same problem. This is because the single stage model presents a myopic framework in which there are no Stage 2 decisions. Since $K_c^* = K_a^* + K_b^*$ for the single stage formulation of the process focused network (as well as the two-stage product plant network), irrespective of cost and demand parameters, there is no risk pooling in the network. This is similar to the arguments found in Kulkarni, Magazine and Raturi (2001). In the two-stage framework, for this set of cost and demand parameters, we find evidence of risk pooling as $K_c^* < K_a^* + K_b^*,\ K_c^* > K_a^*,\ K_c^* > K_b^*$.

	Capacity			Cost
	AC assembly	BC assembly	C	
Process focused network (Section 3.2)	1909.24	496.598	2284.43	32242.40
Product focused network (Section 3.3)	1977.27	541.67	2518.94	28904.40
Single Stage Solution to the Process focused network	1500	300	1800	32850.00

Table 1: Comparing Optimal Solutions for Product and Process Focused Networks

Second, the process focused network acquires less capacity for the common component than the sum of the dedicated capacities. There are advantages of having commonality at the component level. Baker, Magazine and Nuttle (1986) analyze a two-product, two-level model and show that commonality permits a given service level to be attained with a smaller amount of safety stock than would be attainable without commonality. We see similar results here due to the risk pooling in the process focused network at the common component level.

But from our numerical example, it does not seem as though this risk pooling in the process focused network is of any significant value since the product focused network has higher capacity acquisitions but a lower total cost. Indeed, without risk pooling, the cost of the process focused network would be much higher, so there is some value generated by this configuration. The lack of significant risk pooling benefits in the process focused network is due to the relative levels of cost parameters assumed, which render the process focused network worse-off than the product focused network. Thus we study the sensitivity of difference in effectiveness of the two networks for variations in the important cost parameters. We limit these parameters to the common component's capacity acquisition and shipping costs since we capture many of the results by changing the relative values of these two parameters.

4.1 Sensitivity to Common Subassembly Capacity Acquisition Cost

In Figure 5 we vary the cost of common component capacity C_c and plot the ratio of Expected Total Costs of the Product Network to the Process Network. The remaining parameters are fixed at: $C_a = 1$, $C_b = 1$, $\alpha = -12.75$, $\theta = -11.75$, $D_{ac} \sim U(0, 10000)$, $D_{bc} \sim U(0, 8000)$, $p_{ac} = 15$ and $p_{bc} = 15$. In this figure we fix the shipping cost of the common component

(s_c = 0.25). In Figure 5, product focused networks are more effective than process focused networks for small values of common component capacity cost. This is the situation when the cost of shipping common component works against the process focused network. However, as the cost of common component capacity increases, the process focused network becomes the better choice. This is not surprising and reflects the tradeoff between gains due to risk pooling in the process network (reduced overall capacity cost) and the extra shipping incurred. The process focused network overcomes the extra logistics costs of the common component by pooling its capacity.

INSERT FIGURE 5 HERE

What is more interesting is that the advantage of risk pooling at the common component level reaches a maximum, then diminishes and is actually nullified at very high levels of common component capacity cost. We have not observed this type of behavior in the capacity acquisition or the common component inventory literature. Also, this behavior is not easily explained. We conjecture that at very high levels of common component capacity cost (beyond 5.5 in Figure 5) less capacity is ordered in both types of networks, and the penalty costs for shortages become dominant in the objective function wiping out any risk pooling benefits accruing in the process focused network.

4.2 Sensitivity to Common Subassembly Shipping Cost

In Figure 6 we vary the common component shipping cost (s_c) and plot the ratio of Expected Total Costs of the Product Network to the Process Network, as before. We do this at three different levels of common component capacity cost, all in the intermediate range since at very high and very low component capacity costs (see Figure 5), we know that the product plant network dominates. The remaining parameters are fixed at: C_a=1, C_b=1, s_{ac} =2, s_{bc}=3, D_{ac} ~U(0,1000), D_{bc} ~ U(0,8000) p_{ac}=15, and p_{bc}=15. From Figure 6 we see that as the shipping cost for the common component increases, the Product Network becomes increasingly attractive and the preferred choice. The behavior in Figure 6 quantifies an important strategic trade-off; that of risk pooling advantage versus additional logistics costs for Process Focused Networks. We also observe that as the cost of common subassembly capacity increases, the Process Network remains attractive for larger values of shipping costs. This result is the same as that in Figure 5 with the cost of common component in the range 3.5 to 5.5.

INSERT FIGURE 6 HERE

In this section, we have compared product and process focused networks for a wide range of scenarios. The key result is that this decision hinges on the tradeoff between capacity cost of common component and demand side risk pooling and logistics costs. We also learn that risk pooling at the common component level may not always be the best strategy since it is most effectively deployed when the firm has the volumes to sustain it. This requires that the firm not only have high variance in demand but also an intention to serve this demand. In the next section, we derive several analytical results and provide insights into the sensitivity of the process focused network to cost parameters.

5. Sensitivity of Process Network to Cost Parameters

The numerical analysis comparing the product and process focused networks aids us in drawing conclusions about the conditions in which one is preferred over the other. In this section we study the sensitivity of the Process network to cost parameters for two reasons. First, this provides insight into the response of a risk-pooled configuration to changes in the cost and helps us appreciate the importance of estimating cost parameters accurately. For example, Cohen and Moon (1990) demonstrate how changes in cost structure can result in a range of network/supply chain configurations. Second, the joint effects of demand uncertainty and changes in cost coefficients can create unpredictable effects in a risk-pooling environment, which may not be obvious without thorough analytical investigation. Two results relate the sensitivity of the optimal capacity acquisition (K) to the stage 2 cost parameters (α and θ) and the stage 1 capacity acquisition cost (C). We also investigate the sensitivity of the service level (probability that demand for both products is satisfied) to cost parameters.

5.1 Sensitivity of Optimal Capacity Acquisition to the Stage 2 costs

For the process focused network (Figure 2), the two-stage formulation in section 3.2 suggests some non-intuitive conclusions about the sensitivity of the optimal capacity acquired to stage 2 costs. We first propose the sensitivity results and discuss the interpretation of this proposition. We then show numerical illustrations, which demonstrate *substitution-no substitution* effects across the capacity vector similar to our analytical results.

Proposition 1: The sensitivity of the unique optimal capacity acquisition to the second stage composite cost coefficients is given as,

$$\frac{\partial K}{\partial a} = -\begin{pmatrix} \dfrac{q^2[(I_1+I_5)(I_2+I_3)+I_1I_5]}{|H|} & \dfrac{q^2I_3I_5}{|H|} & \dfrac{q^2I_3(I_1+I_5)}{|H|} \end{pmatrix}' \left(P_{\Omega_3}+P_{\Omega_4}\right)$$

$$\frac{\partial K}{\partial q} = -\begin{pmatrix} \dfrac{q^2[I_3I_5(P_{\Omega_1}+P_{\Omega_2})+I_1I_3P_{\Omega_2}-(I_1I_2+I_2I_5+I_1I_5)P_{\Omega_3}]}{|H|} \\ \dfrac{EP_{\Omega_1}+FP_{\Omega_2}-qI_5[(q-a)I_6-aI_4]P_{\Omega_3}}{|H|} \\ \dfrac{\hat{H}P_{\Omega_1}+IP_{\Omega_2}-q(I_1+I_5)[(q-a)I_6-aI_4]P_{\Omega_3}}{|H|} \end{pmatrix}$$

where |H|, E, F, $\hat{H}$, I > 0 and $I_i > 0$ ∀ I,

P_{Ω_i} = vector of probability measure of demand domains,

K is the optimal capacity acquisition vector, and,

α is the composite cost coefficient in Stage 2 that represents the shipping and penalty cost differential for product AC ($\alpha = s_c + s_{ac} - p_{ac}$) and $\alpha/\theta > 1$.

$$\left.\begin{array}{l} I_1 = \displaystyle\int_0^{K_c-K_b} f(x,K_b)\,dx \ , \ I_4 = \int_0^{K_c-K_a} f(K_a,y)\,dy \\ I_2 = \displaystyle\int_{K_c-K_b}^{K_a} f(x,K_c-x)\,dx \ , \ I_5 = \int_{K_b}^{\infty} f(K_c-K_b,y)\,dy \\ I_3 = \displaystyle\int_{K_a}^{\infty} f(x,K_c-K_a)\,dx \ , \ I_6 = \int_{K_c-K_a}^{\infty} f(K_a,y)\,dy \end{array}\right\}$$

Here "x" and "y" are the demand realizations (*observed demand*) for products AC and BC and "Φ" denotes the distribution function of the joint probability density function of demand ("f"), of products AC and BC.

According to Proposition 1, the optimal capacity acquisitions are monotonically decreasing in α. This result is not surprising: as the shipping cost of the common subassembly or AC increases or the penalty cost of product AC decreases, we have an overall decrease in the capacity acquired for assembly of products AC and BC, as well as for manufacturing the common component. Further, we know that $\left|\frac{\partial K_a}{\partial a}\right| > \left|\frac{\partial K_c}{\partial a}\right| > \left|\frac{\partial K_b}{\partial a}\right|$. This is intuitive since we are investigating sensitivity with respect to α, and we expect the maximum sensitivity to be associated with product AC.

The optimal capacity acquisitions for the assembly of product BC and that for the common component are monotonically decreasing in θ, but the same cannot be said of the optimal capacity acquisition for the assembly of product AC. Recollect that θ is the composite cost coefficient in Stage 2 that represents the shipping and penalty cost differential for product BC ($\theta = s_c + s_{bc} - p_{bc}$) and that $\alpha/\theta>1$. We see that $\frac{\partial K_a}{\partial q}$ cannot be signed in general. This would mean that as the shipping cost (for common component or for BC or for both) increases, or as the penalty cost (p_{bc}) decreases, we would have a decrease in K_c and K_b, but we may have an increase in K_a. This may seem counter-intuitive, but can be explained on the basis of the domain probabilities and the corresponding shadow prices.

From Proposition 1 we see that the sign of the cost sensitivity term for K_a is dependent upon the relative values of probabilities of demand domains 1, 2 and 3 from Figure 4. Thus, when the optimal capacity acquisitions are such that $P_{\Omega_3} >> P_{\Omega_1}$ and $P_{\Omega_3} >> P_{\Omega_2}$, then it may prove beneficial to *substitute* some of the common component capacity for product AC (increase K_a). An example clarifies this. In the regions considered here, Ω_3 dominates, or there is a very high likelihood that product AC is constrained by capacity ($D_{ac} > K_a$). A decrease in the penalty cost for product BC would result in a decrease in capacity acquired for BC and C. However, since AC production is constrained by capacity, capacity acquired for AC might increase. Table 2 clarifies the before and after situation. As can be observed from Table 2, the proportionate share of common component for assembling AC has increased, demonstrating the substitution effect (see Proposition 4 of Van Mieghem (1998) and Fine and Freund 1990).

	Capacity for AC assembly (μ_{AC}=1500) ($D_{ac} > K_a$)	Capacity for BC assembly (μ_{BC}=500)	Capacity for C production ($D_{bc} > K_c - K_a$)	Max. proportion of C production reserved for AC v/s BC
Before	1909	496	2284	0.83 v/s 0.21
After	1918	214	1977	0.97 v/s 0.11

Table 2: Demonstrating the Substitution Effect when Penalty Cost for BC decreases

The pooling of the common component production in a single plant provides us the *option* of taking advantage of implicit profits, by disproportionately reducing K_c compared to K_b and using this (now "released") common component capacity to assemble more of product AC. If the common component were to be manufactured in two separate plants, then this effect would not be observable. Also, if the demand for AC would be relatively small compared to K_a, then again such an effect would not be observed. Similar to an observation by Van Mieghem (1998), in our case the pooling of the common component capacity provides the *optimal cost/benefit response* to demand variability and is dependent on the entire shape of the demand distribution.

The composite cost coefficients (α and θ) allow an assessment of the joint impact of a variation in s_c on the optimal capacity acquisitions. This is given by the sum of the appropriate cost sensitivity terms in Proposition 1. It is interesting that K_a is then always monotone decreasing in s_c (i.e. $\frac{\partial K_a}{\partial a} + \frac{\partial K_a}{\partial q} < 0$). This means that although some of the common component capacity is substitutable, this substitution is not complete [see proposition 3 in Van Mieghem (1998)].

Figure 7 presents a numerical illustration of Proposition 1. The parameter values are: $C_a = 5$, $C_b = 3$, $C_c = 2.5$, $\alpha = -20$, $D_{ac} \sim U(0, 3000)$, $D_{bc} \sim U(0, 1000)$. In Figure 7 we plot the optimal capacity acquisitions against the change in θ. As predicted by Proposition 1, the capacity acquired for the assembly of product BC decreases, the common component capacity acquired decreases, however the capacity acquired for assembly of product AC, *increases*, although very slightly.

INSERT FIGURE 7 HERE

5.2 Sensitivity of the Optimal Capacity Acquisitions to Cost of Capacity.

We expect the amount of capacity to be a decreasing function of the cost per unit to acquire that capacity.

Proposition 2: The sensitivity of the unique optimal capacity acquisition to the marginal capacity acquisition costs is given as,

$$\frac{\partial K}{\partial C} = -\frac{1}{|H|}\begin{pmatrix} q^2[(I_1+I_5)(I_2+I_3)+I_1I_5] & q^2I_3I_5 & q^2I_3(I_1+I_5) \\ q^2I_3I_5 & \begin{matrix} -q\,[(q-a)I_6-a\,I_4](I_2+I_3+I_5) \\ +q^2I_3(I_2+I_5) \end{matrix} & -q\,I_5[(q-a)I_6-a\,I_4-q\,I_3] \\ q^2I_3(I_1+I_5) & -q\,I_5[(q-a)I_6-a\,I_4-q\,I_3] & -q(I_1+I_5)[(q-a)I_6-a\,I_4-q\,I_3] \end{pmatrix}$$

where K is the vector of capacity and C is the vector of costs associated with each of the three plants in the process focused network.

In Proposition 2 we see that *all* optimal capacities are strictly decreasing in the marginal capacity acquisition costs. This result is interesting since it is different from the effect observed by Fine and Freund (1990) in Theorem 2 and by Van Mieghem (1998) in Proposition 3. This is because there is *no substitution* present between common component capacity and the dedicated capacities. This result is important for two reasons. One, it shows that the substitution effect observed earlier is present with respect to operational cost parameters and not the strategic ones. Second, and perhaps more important, in conjunction with Proposition 1 it shows that the *option* provided by pooling common component production may not be immediately evident unless the strategic configuration decision explicitly considers average operational costs (that is, costs associated with second-stage decisions).

We illustrate Proposition 2 in Figure 8 by plotting the changes in optimal capacity acquisitions with increasing marginal cost of common component capacity. Parameter values are assumed to be: $C_a = 5$, $C_b = 3$, $\alpha = -20$, $\theta = -10$, D_{ac}~U (0,3000) and D_{bc}~U(0,1000). Similar graphs can be constructed for demonstrating the sensitivity of capacity acquired to increases in C_b and C_c.

INSERT FIGURE 8 HERE

5.3 Sensitivity of Joint Service Level to Stage 2 Cost Parameters

Firms are concerned about the service level provided in a process plant network. We provide a numerical illustration, which is of interest from the perspective of service levels in both markets. Recall that the probability (P_{Ω_0}) of domain Ω_0 in Figure 4 represents the chance that demand for both products is *completely satisfied*. We plot P_{Ω_0} whilst varying θ (= $s_c + s_{bc} - p_{bc}$) in Figure 9. Observe that the probability of jointly providing 100 % service levels for both markets decreases as θ increases.

INSERT FIGURE 9 HERE

This behavior is interesting especially in conjunction with Figure 7. As θ increases from -10 to -6.5, we acquire less common subassembly capacity and capacity for product BC, but more capacity for product AC. This follows from Proposition 1. This would mean that we would expect product AC to have 100% service level more often. However this increase in service level is more than offset by the decrease in service level for product BC. Hence the overall probability of providing 100% service for both products is reduced with the increase in θ. We observe that in a process plant network with two markets, decreasing penalty costs of shortage in one market (an decrease in p_{bc} in this case) leads to a decrease in the aggregate service level even though there is no correlation between the demands for AC and BC.

6. Conclusions and Future Research

We have presented analytical models for gaining insight into the strategic plant configuration problem. Specifically we contrast the product and the process focused networks under demand uncertainty. The two-stage optimization model developed minimizes the cost of capacity, logistics and penalties associated with not meeting demand. Several results characterize the properties of the optimal solution as well as the sensitivity of the optimal decisions to variations in cost parameters.

We show with a numerical example that the risk pooling advantage offered by a process focused network is mitigated at small or large values of common subassembly capacity costs. Firms would prefer product-focused networks at very low and very high cost of the capacity for the manufacturing the common component. When the cost of common component capacity is very low, product focused networks are preferred because of savings due to reduction in (elimination of) shipping cost. In intermediate cost ranges, process-focused networks are preferred as risk pooling generates benefits with consolidation of common component manufacturing. These benefits are nullified at very high cost of capacity as firms choose to not satisfy all of the market demand – once again product- focused networks are preferred here. Product focused networks are also preferred over process focused networks when the cost of shipping common components in the manufacturing network is high. As the cost of capacity for the common component increases within some range, this advantage is diminished.

We have investigated the sensitivity of optimal capacity acquisitions in the process focused network to cost coefficients through analytical reasoning as well. We find that an increase in logistics costs (or decrease in the penalty cost) may at times cause an increase in acquisition of subassembly capacity for the more attractive product. This result is non-intuitive and interesting from the perspective of strategic planning. This result validates the notion of unequal substitution found in the literature. It also suggests that generic rules such as “always decrease capacity when cost of capacity increases” or “always decrease capacity when penalty cost decreases” may not apply for manufacturing networks. In this case we find that a decrease in the penalty cost of one product may well result in an increase in capacity for another product.

Convolution of factors such as uncertain demand and cost mix differentials can render the two-stage strategic plant configuration problem quite difficult. Our models though admittedly stylized provide several opportunities for further research. First, our models and comparison could be extended to the multi-period, multi-product case with inbound and well as outbound logistics and inventory. As suggested in Van Mieghem and Rudi (2002) we expect our results and insights to extrapolate to a dynamic setting, but such an investigation may prove to be an important extension. Second, one could compare the models presented in this paper and their extensions in the presence of a fluctuating demand and supply environment. For example, demand/supply may be represented by a Markov modulated Poisson process. It is not immediately clear how the models considered in this paper would compare in such a situation.

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Figure 1: Multi-plant network of a hard-disk drive manufacturing firm

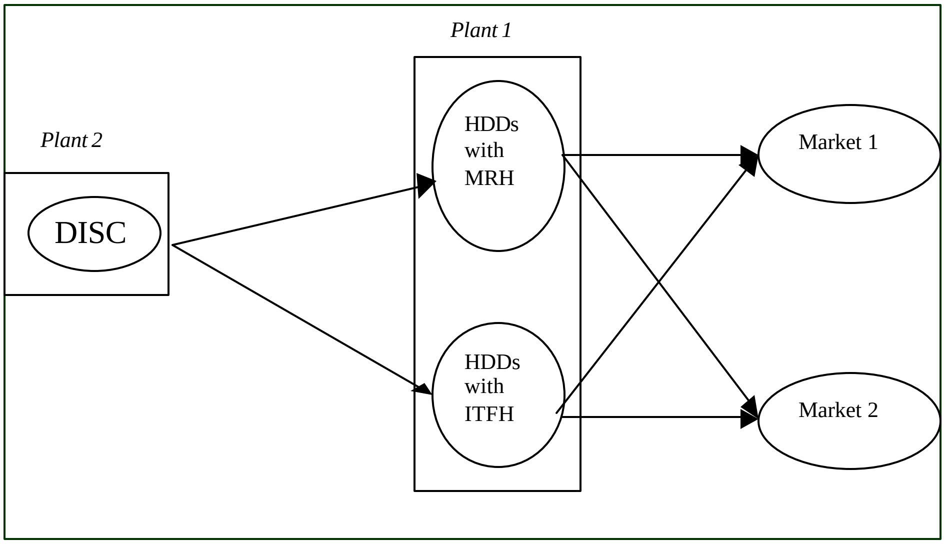


Figure 2: Process Focused Network with component commonality
Common component is made in a dedicated plant

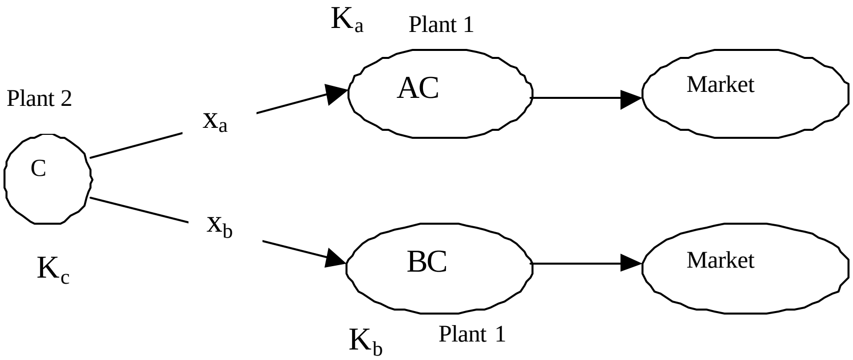
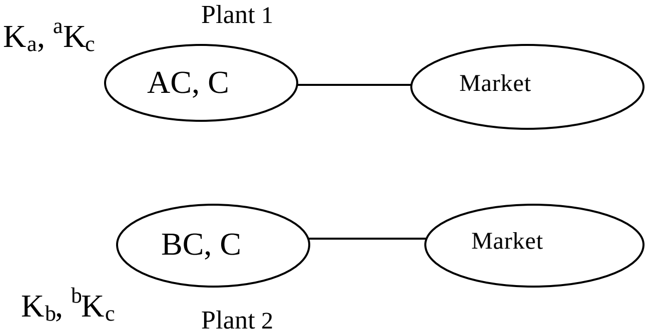


Figure 3: Product Focused Network with component commonality
Common component is made in assembly plants



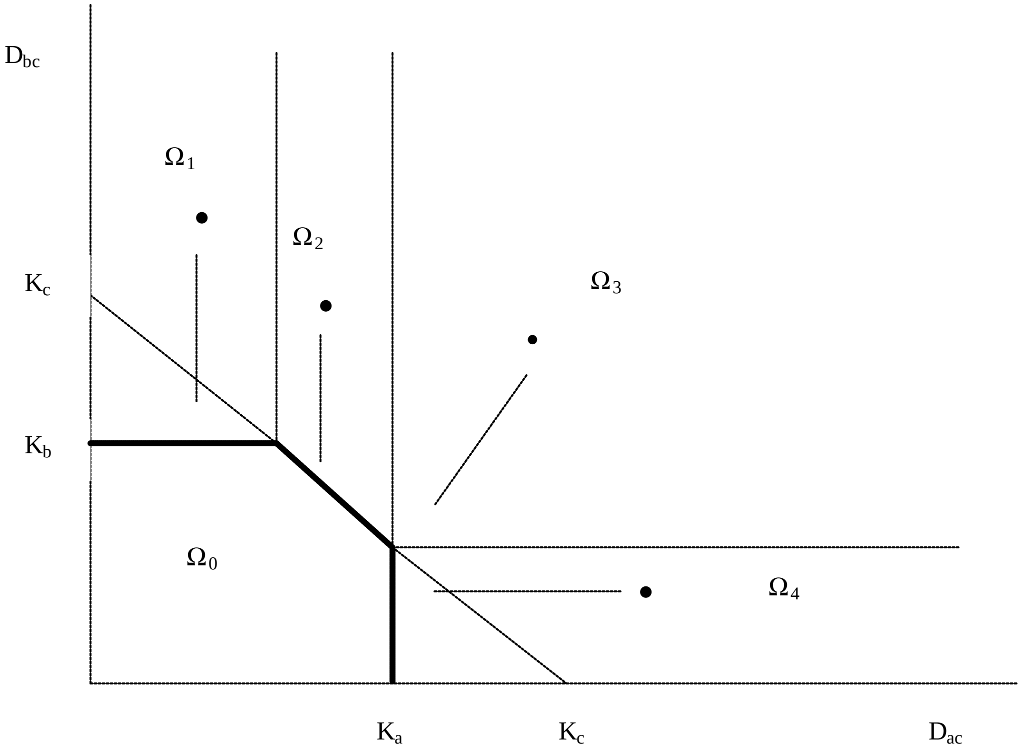


Figure 4: Demand Vector Space Partition

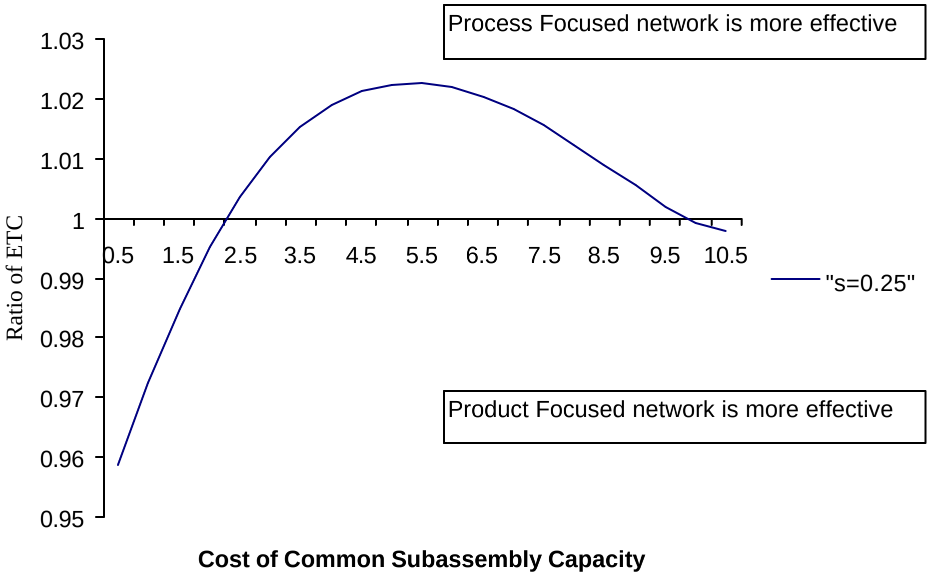


Figure 5: Optimal Costs for *Product versus Process* Focused Networks over varying cost of capacity for the common component

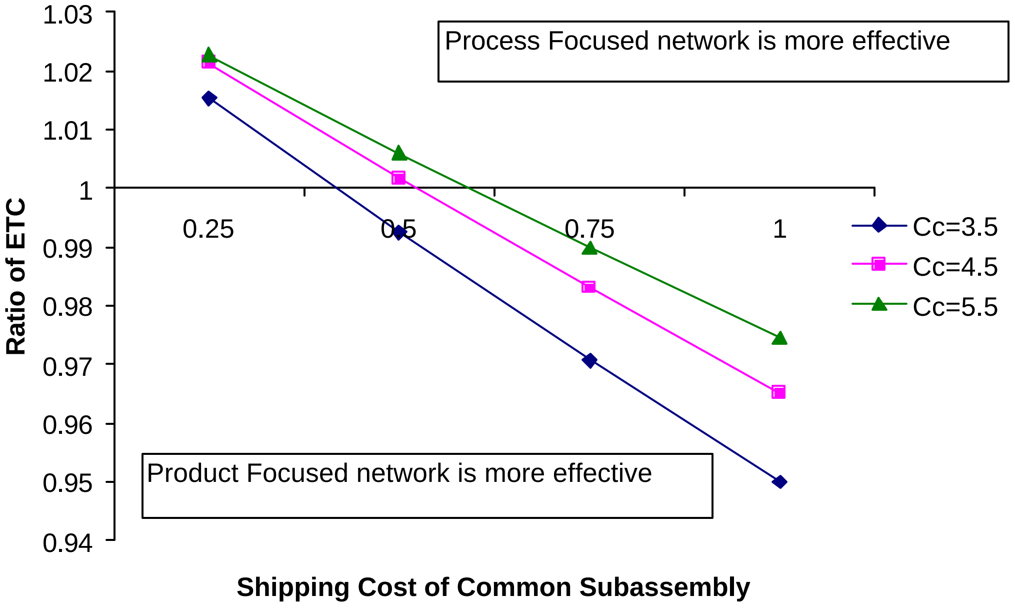


Figure 6: Optimal Costs For *Product Versus Process* Focused Networks Over Varying Cost Of Shipping For The Common Component

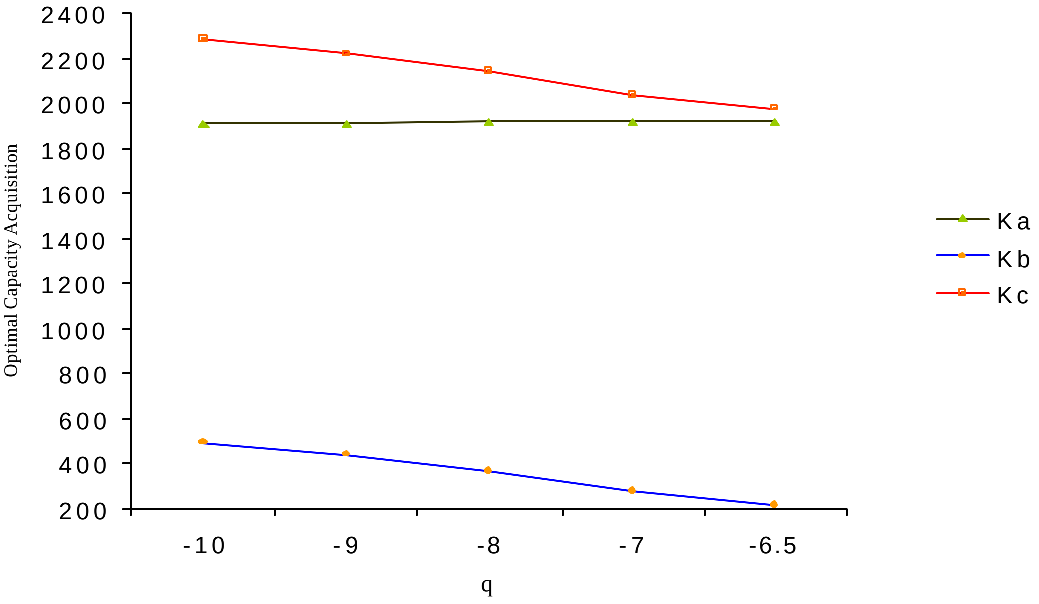


Figure 7: Sensitivity of Optimal Capacity Acquisitions to Shipping and Penalty Cost Differential

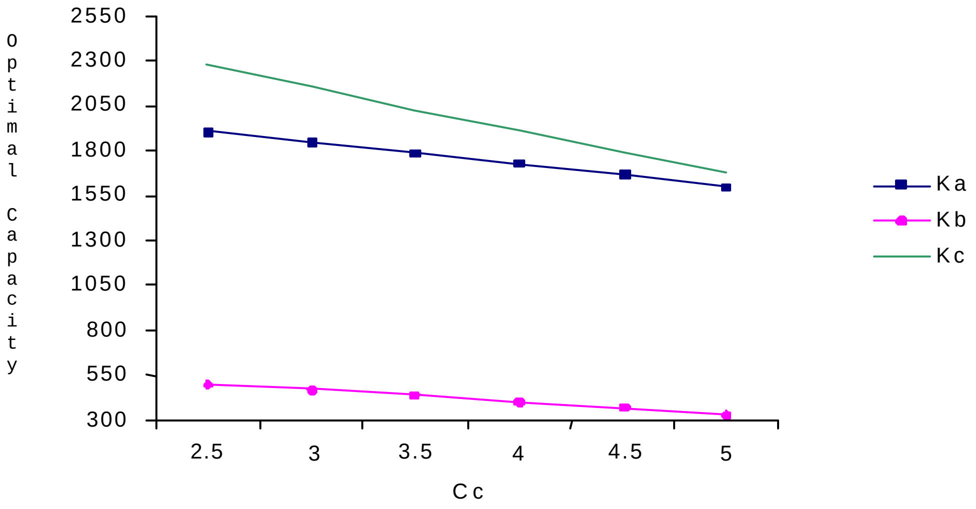


Figure 8: Sensitivity of Optimal Capacity Acquisition to Cost of Capacity for Common Component

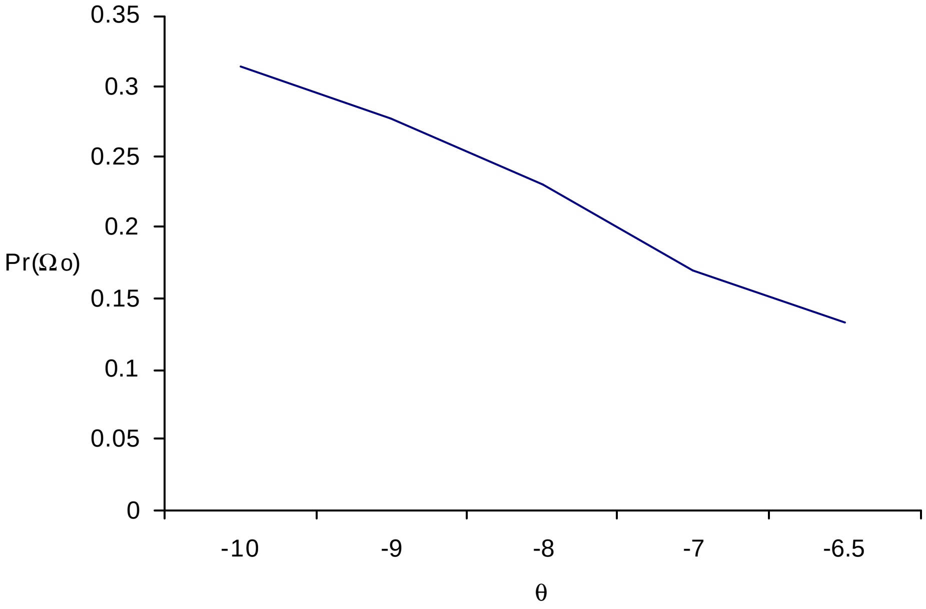


Figure 9: Sensitivity of Joint 100% Service Level to Shipping and Penalty Cost Differential ($q = s_c + s_{bc} - p_{bc}$)