

Seeking Closure: Competition in Complementary Markets

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Abstract

A firm with a product that competes in a market that has a complementary product (in a different market) must consider the interdependence between the complementary products as well as the competition within markets. If the firm participates in both markets, the balancing act becomes even more challenging. This paper provides insights about strategies in this latter setting. When should the firm seek to keep its products closed to competitor's complementary products, and when would the firm be better off by accepting a common standard? To address these questions, we employ standard game theoretic analysis to a simple spatial model that captures aspects of both inter-market externalities and intra-market competition. We find that if a firm participates in both markets and chooses a closed standard, it achieves lower profits compared to an open standard, but greater market share. Surprisingly, we find that customers are better off when standards are kept proprietary.

Consider Apple Computer's sales of iPod players and iTunes music. The iPod competes in the market for MP3 players, and iTunes competes in the complementary market for downloadable songs. The demand for the MP3 players (hardware) and for the music available for the players (software) each influence demand for the other. By adhering to a proprietary technological standard, Apple attempts to reinforce such cross-market externalities between its own product offerings – to the chagrin of its competitors.

Keywords: *supply chain management, complementary markets, proprietary standards*

1. Introduction

In the summer of 2004, RealNetworks released software (ironically called “Harmony”) as part of its music player RealPlayer 10.5 that allowed music buyers to purchase from Real’s online store music that worked on multiple DRM formats.¹ With this software, buyers could now buy songs that played not only on non-Apple MP3 players (as before), but also on Apple Computer, Inc.’s MP3 player, the iPod. Apple quickly expressed displeasure. “We are stunned that RealNetworks has adopted the tactics and ethics of a hacker to break into the iPod,” the company declared in a press release. Apple further warned its customers that “when we update our iPod software from time to time it is highly likely that Real’s Harmony technology will cease to work with current and future iPods” (Macnewsworld 2004).

A firm with a product that competes in a market that has a complementary product (in a different market) must consider the interdependence between the complementary products as well as the competition within markets. If the firm also participates in the market for the complementary product, the balancing act becomes even more challenging. This paper provides insights about strategies in this latter setting, such

¹ Music distributed on the Internet predominantly uses a standard, abbreviated as MP3, that was developed by the Moving Picture’s Expert Group for coding audio visual information in a digital compressed format. Music websites combine the MP3 standard with Digital Rights Management (DRM) to protect the intellectual property of the artists. The use of different DRM formats raises the possibility/reality of incompatibilities between MP3 players and the various music websites. It is possible for savvy consumers to convert MP3 files from one DRM format to another, but the process is awkward and time consuming.

as Apple Computer's complementary iPod players and iTunes music versus their competitors in the markets for MP3 players and for downloadable songs. The availability of hardware (MP3 players) and software (music available for the players) each influence demand for the other market.

It is interesting to note that Apple did not embrace the opening of a market that provided a positive externality to its iPod customers, who now had more choices for their music purchases. Ignoring the fact that Real's actions made the iPod more attractive, Apple instead chose to fight against RealPlayer as a competitor to iTunes.

Was Apple's resistance to an open standard for music appropriate, or was it misguided? When should a firm make its technology exclusive to its own standard, and when should it open up? Assuming a firm is seeking closure, which side of the two-sided market should be exclusive? For example, should Apple design iPod to play only songs purchased from iTunes and/or should the iTunes shop sell files that can be played only on the iPod? How are these dynamics affected by the relative attractiveness of the firm's products vis-à-vis competitor's products?

In 2005, the iPod family owned 73% of the market for MP3 players and iTunes owned 75% of the market for downloading (legal) music (Leonard 2006). How is market share related to the attractiveness of a closed market?

The problem of competition across complementary markets can also be of interest to lawmakers, as highlighted by Gilbert and Riordan (1995), Evans (2002), Gandal (2002), and others. What is the effect of a proprietary standard on consumers, as measured by consumer surplus?

To address our research questions, we use a spatial model in a game theoretic setting. There are two complementary markets, each featuring two products. Although the framework could be used more generally to analyze other scenarios, we analyze the specific scenario where one firm participates across both markets with complementary products of one standard. In each of the two markets, the firm faces a provider of a competing product. The two competitors act independently, but use an open standard. In contrast, the firm participating in both markets chooses whether to keep its standard closed (proprietary) in one or both of the markets, or to use the open standard. (The competitors are willing to have the standard be open to the one firm.) We assess pricing strategies and profitability when the markets for complementary products are open (i.e., the one firm uses the open standard) and when they are closed.

We find that if the firm participating in both markets chooses a closed standard, it achieves lower profits, lower prices, and higher market share compared to using an open standard. Together with lower prices, maintaining a high market share under a closed standard provides barriers to entry, closing a large part of the market to possible entrants. Customers under closed standards are captive to that standard, and any future purchases must remain with the same firm if customers want to retain the benefit from earlier purchases in the complementary market of that standard. While the desire for a high market share and enhanced barriers to entry might explain a firm's use of a proprietary standard, we also surprisingly find that consumers may actually be better off under closed standards.

We structure this paper as follows. In section 2 we review the related literature. We introduce our model in section 3. Section 4 contains our analysis and results. We conclude in section 5.

2. Literature Review

Our work builds on a substantial stream of research where the availability or sale of one product affects the utility that another customer can derive from the consumption of another product. Such externalities can occur in various contexts, and some important related areas of research are associated with terms like network externalities, multi-product pricing (bundling / tying), two-sided markets or product complementarity.

The research on (direct) network externalities (e.g., Katz and Shapiro 1985, Farrell and Saloner 1986) considers settings where the entry of a new customer to the network increases the utility that all other customers can derive from participating. A cell-phone network, where each new entrant increases the number of people that can communicate via the network, is a typical example for this phenomenon. On the other hand, work on multi-product pricing or two-sided markets (e.g., Armstrong 2002, Rochet and Tirole 2003 and 2004, Hagiu 2004, Farhi and Hagiu 2004, Reisinger 2004, Parker and Van Alstyne 2005) considers problems where the increase in sales or availability of a product alters the utility that customers associate with a different product.² Similar to

² The key difference between multi-product pricing and pricing in two-sided markets is that in the former the same consumer internalizes the externalities between the matched markets (e.g., razor and razor blades), whereas different groups of customers are assumed to exist in the latter case (e.g., software developers and customers).

most of the literature that applies these models to the case where positive externality between markets is derived from product complementarity (e.g., Bhaskaran and Gilbert 2005, Parker and Van Alstyne 2005, Heese and Pun 2006), in our model the two markets might present the same set of customers. But we do not require that a consumer who purchases a product of a certain standard in one market buy a product of this standard in the matched market. Consumers are free to choose their preferred product in either market; however, other things equal, matched products provide the higher utility. In contrast to the above articles, where the product compatibility structure is taken as exogenous, our work specifically addresses the question of when a firm might prefer proprietary to open standards.

Research that considers the issue of compatibility in markets for product systems usually focuses on how compatibility (or not) affects the equilibrium market structure or product variety (Economides 1991, Economides and Flyer 1997, Caillaud and Jullien 2003, Kang et al. 2004). Katz and Shapiro (1994) provide a qualitative discussion of systems competition between product systems. Unlike our paper, most of this research assumes a symmetric ownership structure, where the providers of complementary products are all either vertically integrated or independent. Kang et al. (2004) derive conditions for an asymmetric structure in equilibrium, but in their model product systems are always incompatible.

Church and Gandal (2000) consider two markets for complementary products (two types of hardware and two types of software), and study the incentives of a hardware provider to vertically integrate with the matched software firm, and then to make the product system incompatible to the competitors. Besides specifying conditions

when such behavior is an equilibrium, Church and Gandal also discuss welfare implications of such closure. While closely related to our work, their research differs from ours in several important aspects. First, they assume one-way externality, such that software variety benefits hardware sales, but not vice versa. Also, while their hardware market is differentiated and based on a Hotelling-type model (Hotelling 1929), their software market is not differentiated. The role of the software market as a support for the hardware market is also captured in their sequential game structure, where hardware prices are set before software prices. We assume more equivalent markets that mutually support each other (two-way externality). Customers have preferences for each product type, and we assume that prices in the two markets are set simultaneously. Like Church and Gandal, we find that closure leads to lower prices, since the integrated firm internalizes the externality across markets. However, while closure always decreases consumer welfare in their model, we find that customers might actually benefit from proprietary standards.

3. The Model

Assume there are two markets for complementary product types 1 and 2 (indexed with m). Assume these markets are of equal size³, and without loss of generality, we normalize them to size one. There are competing providers in each market. Similar to

³Our model can be extended here to capture differences between market sizes, and later to capture differences in costs and consumer preferences. The resulting expressions for equilibrium solutions in these more general settings are unwieldy. We focus on the simplified model to derive analytic insights into the problem of interest.

Armstrong 2002, we model complementarity by assuming that sales of a product in one market has a positive effect on the utility that customers derive from a compatible product on the other market. For example, consider the case where there are two different standards A and B (indexed with s), such that each of the two products in market 1 can only be used with the corresponding product (i.e., that has the same standard) in market 2. In this case, a sales increase of the product with standard A in market 1 will increase the utility that customers in market 2 derive from owning a product of standard A .

We assume that the customers in each of the two markets are heterogeneous in their preferences with respect to the two providers, and we use a Hotelling-type model to derive the respective market shares. (See Lancaster 1990 for a discussion of the application of Hotelling models in non-spatial contexts.) In particular, customers' preferences for their "ideal product" in each market are uniformly and independently distributed along a line segment of unity length. Given our interest in the competitive dynamics in such a setting as well as the complexity of the problem and the model, we simplify the analysis by beginning with an assumption of full market coverage. (As a consequence, in this section we use the terms sales and market share interchangeably.) In addition, we assume that the offered products perfectly fit the customers at the extreme points of the markets, represented as the end points of the line segments. All other customers incur a disutility corresponding to the position on the line of their ideal point versus the end point. In section 4.3 we extend this model to consider the case where customer ideal points extend infinitely beyond the firms' products' positions such that there is not full market coverage, and then pricing affects total industry sales.

Let α denote the strength of the externality effect between the two markets, in either direction, and assume that a customer in market m has a base reservation price R_m for a unit of product, before considering the externality from the complementary product. For ease of exposition, we use \bar{m} to denote the market that is complementary to market m (i.e. $\bar{m} = 3 - m$). Similarly, define \bar{s} as the standard that competes with standard s (i.e. $\bar{s} = \{A, B\} \setminus s$).

The positive externality from the market of the complementary product increases the customers' utility for the compatible product. For example, if firm A 's standards are *closed* and A has sales of S_{A2} in market 2, then the reservation price for the product of standard A in market 1 is increased by αS_{A2} , to $\hat{R}_{A1}^{closed} = R_1 + \alpha S_{A2}$. If the product of standard A in market 1 is *open*, i.e. if it is compatible to all products of type 2, its reservation price increases to include the positive externality of having a wider compatible offering. In that case, the adjusted reservation would be

$\hat{R}_{A1}^{open} = R_1 + \alpha(S_{A2} + S_{B2}) = R_1 + \alpha$. The adjusted reservation prices for all possible scenarios are given in the appendix.

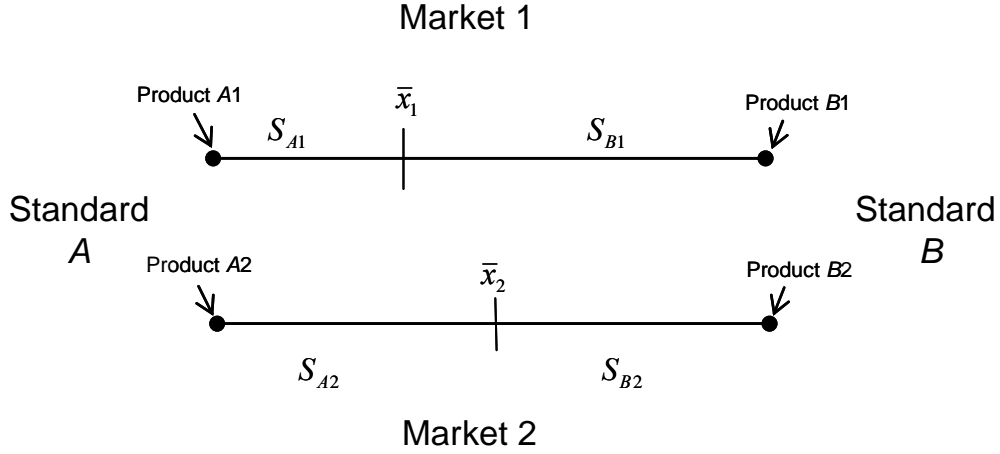


Figure 1: Distributions of Customer “Ideal Products” in Complementary Markets

Figure 1 shows the line segments representing the “ideal products” for customers in market 1 and market 2. For customers not at the end points, the product characteristics do not perfectly match the different individual taste preferences. In either market, a customer with preference x for the attribute incurs a disutility tx if consuming a product of standard A , and a disutility $t(1-x)$ if consuming a product of standard B . Hence, a customer in market m with preference x associates the utility $U_{Am}(x) = \hat{R}_{Am} - P_{Am} - tx$ with the purchase of a product of standard A at retail price P_{Am} and

$U_{Bm}(x) = \hat{R}_{Bm} - P_{Bm} - t(1-x)$ with the purchase of a product of standard B at retail price P_{Bm} . The equality $U_{Am}(\bar{x}_m) = U_{Bm}(\bar{x}_m)$ determines the indifferent customer \bar{x}_m in market

m . Our interest lies on interior point solutions with competition between the two standards and where each product captures some sales. Consequently, in the following we restrict our analysis to the region where $0 < \bar{x}_m < 1$. (Conditions that ensure this are given in the appendix.) Since $S_{Am} = \bar{x}_m$ and $S_{Bm} = 1 - \bar{x}_m$ (cf. Figure 1), the location of the

indifferent customer in market m is directly associated with the sales S_{sm} of standard s in that market. In general,

$$S_{sm} = \frac{1}{2} + \frac{(\hat{R}_{sm} - P_{sm}) - (\hat{R}_{\bar{s}m} - P_{\bar{s}m})}{2t}. \quad (1)$$

Note that for any product, the adjusted reservation price \hat{R}_{sm} on the right hand side of equation (1) depends on the share of the complementary market that is compatible to this product. Hence, choices regarding the use of proprietary standards directly affect sales of all products. Using the specific reservation price adjustments for a given compatibility structure (see appendix), explicit expressions for sales can be obtained by simultaneously solving equation (1) for all four products. Assuming a per-unit production cost of C_m , profits associated with sales of a product of type m and standard s are

$$\pi_{sm} = (P_{sm} - C_m)S_{sm}.$$

We generally assume that the product providers set retail prices simultaneously and solve for the resulting Nash Equilibrium. The ownership structure of the markets determines the optimization objectives. In this paper, we focus on the case where one firm controls products associated with standard A , while the providers of the competing standard B are independent. Thus, the prices P_{A1} and P_{A2} will be set to maximize joint profits $\pi_A = \pi_{A1} + \pi_{A2}$, whereas the providers for the two products associated with standard B each independently set their respective prices P_{Bm} to maximize their individual profits π_{Bm} .

We take the perspective of the owner of standard A (firm A) who participates in both markets. We assume that the providers of products $B1$ and $B2$ support open

standards and that it is for firm A to decide whether to join that standard or whether to maintain a closed proprietary standard.⁴ We determine the effect of A 's decision on profits, prices, and sales.

4. Analysis & Results

We begin our analysis with a base model of identical firms and comparable products and focus on the differences arising from the different ownership structures. In section 4.1 we use the base model to show how firm A is able to internalize and capitalize on the cross-market externality by jointly optimizing its pricing in both markets while firms $B1$ and $B2$ must independently maximize profits of their respective products. In section 4.2, we extend the base model to capture a customer preference for one product, introducing an additional asymmetry that allows differences in customer preferences (reservation prices) between products in one market. In section 4.3, we consider an extension of the base model that relaxes the assumption of full market coverage. Specifically, in order to capture both competition between firms as well as price-elastic market size, we assume that markets expand beyond the products' locations.

Across all models, we assume that it is Firm A that chooses whether to keep markets 1 and 2 closed. If firm A chooses a closed market, the products using standard A are incompatible with products using standard B . Firm A can choose to keep both markets closed, open market 1 only, open market 2 only, or open both markets 1 and 2.

⁴ For simplicity, we will denote the firm that provides the product Bm as firm Bm .

4.1 Capitalizing on Ownership Structure

Given the generally symmetric structure across both markets and firms, any differences in profits, prices, and sales of the two standards arise solely from the ownership structure: Firm A participates across both markets while firms $B1$ and $B2$ operate autonomously in markets 1 and 2, respectively. Since firm A has ownership across both markets, it is able to internalize the complementarity, such that resulting profits, prices, and sales are not necessarily symmetric.

We make the following assumptions that ensure the existence of a unique equilibrium.

$$t > \alpha \quad (\text{Assumption 1})$$

Specifically, Assumption 1 assures (joint) strict concavity of the optimal price response functions, ensuring the existence of a unique price equilibrium. Intuitively, this condition implies that customer fit requirements – how closely the product matches the customer’s ideal product within a market – are more significant than cross-market externalities. A different model would be required for situations where cross-market externalities dominate.

We first compare profits based on A ’s decision to open one or more markets. Denote resulting profits for A as π_A^{open} , π_A^{open1} , π_A^{open2} , and π_A^{closed} , corresponding to A ’s decision to open both markets, market 1 only, market 2 only, or to close both markets, respectively. Similarly, denote resulting profits for firm Bm as π_{Bm}^{open} , π_{Bm}^{openm} , $\pi_{Bm}^{open\bar{m}}$, and π_{Bm}^{closed} , where the superscript $openm$ denotes the case where A decides to open m

(the market in which firm Bm competes) and $open\bar{m}$ corresponds to A 's decision to open the complementary market for firm Bm , market \bar{m} .

Proposition 1 establishes the effect of A 's decision on each firm's profits.

(Equilibrium prices and all proofs are given in the Appendix.)

Proposition 1:

- a. $\pi_A^{open} > \pi_A^{open1} = \pi_A^{open2} > \pi_A^{closed}$ and $\pi_{Bm}^{open} > \pi_{Bm}^{open\bar{m}} > \pi_{Bm}^{openm} > \pi_{Bm}^{closed}$.
- b. $\pi_A^{open} = \pi_{B1}^{open} + \pi_{B2}^{open}$, $\pi_A^{closed} > \pi_{B1}^{closed} + \pi_{B2}^{closed}$, and $\pi_A^{openm} > \pi_{B1}^{openm} + \pi_{B2}^{openm}$.
- c. $\frac{\pi_A^{open} - \pi_A^{closed}}{\pi_A^{closed}} < \frac{\pi_{Bm}^{open} - \pi_{Bm}^{closed}}{\pi_{Bm}^{closed}}$.

Proposition 1a states that profits will be higher for firm A , as well as for firms $B1$ and $B2$, if A chooses an open standard. If firm A opens only one market, its competitor in that market is more profitable compared to when A chooses a closed standard, but not as profitable as A 's competitor in the other market which now has access to the open complementary market. Neither competitor is as profitable as when A chooses open standards in both markets. Generally, a market with an open standard in the complementary market is able to capture the cross-market externality effect from the entire market rather than just the part of the complementary market with the compatible standard. Being conscious of the interactions between the two markets, under closed markets firm A has strong incentive to reduce prices, in order to gain market share and harness the mutually reinforcing externalities. This pursuit of market share can lead to very competitive pricing as we will confirm below. This strategic role of such

externalities between markets is most pronounced when standards are closed in both markets, and Proposition 1a shows that this case leads to the lowest profits for all firms.

Propositions 1b and 1c compare profitability across firms, based on A 's decision. Under a closed standard we saw that firm A 's profits were lower than under an open standard, and profits for both firms with standard B were lower than under an open standard (Proposition 1a). Proposition 1c provides an additional insight: when firm A chooses closed standards, the competitors suffer more from the closed standard than firm A does; the percentage increase in profits from opening standards is greater for either firm $B1$ or $B2$ than for firm A .

Open standards allow all three firms to capture externality benefits from the entire complementary market. Under open standards, firm A has no relative advantage. In contrast, under closed standards, only firm A is able to internalize externality effects from the complementary markets and firm A 's profits exceed the sum of the competitors' profits. All firms benefit if firm A opens one or more markets, but the competitors gain more than firm A . Specifically, if firm A chooses an open standard in both markets, firm A 's profits no longer exceed, but are equal, to the sum of the profits of the two competitors $B1$ and $B2$.

We next analyze the effect of A 's decision on pricing. Denote the equilibrium retail prices as P_{sm}^{open} , P_{sm}^{openm} , $P_{sm}^{open\bar{m}}$, and P_{sm}^{closed} , corresponding to A 's decision to open both markets, market m only, market \bar{m} only, or to close both markets, respectively. We determine in Proposition 2 how A 's decision affects each firm's prices.

Proposition 2:

- a. $P_{Am}^{open} > P_{Am}^{openm} > P_{Am}^{open\bar{m}} > P_{Am}^{closed}$ and $P_{Bm}^{open} > P_{Bm}^{open\bar{m}} > P_{Bm}^{openm} > P_{Bm}^{closed}$.
- b. $P_{Am}^{closed} < P_{Bm}^{closed}$, $P_{Am}^{open} = P_{Bm}^{open}$, $P_{Am}^{open\bar{m}} < P_{Bm}^{open\bar{m}}$, and $P_{Am}^{openm} > P_{Bm}^{openm}$.

Proposition 2a states that prices will be higher under an open standard versus a closed standard. The product under the open standard is more attractive to customers, and they are charged a higher price. We will see later that the net result of open standards is lower market share for firm A . If firm A opens only one market, prices in both markets for firm A are higher than in the closed case with the higher of the two prices in the particular market that is open. Firm A 's product in the open market is more attractive to customers as it is now compatible with the competitor's complementary product.

In determining prices, firm A needs to consider the involved inter-market dynamics in the trade-off between market share and profit. For instance, if firm A opens market 1, a benefit accrues to the competing product $B2$ in the complementary market. Hence, a focus on bolstering profit margins for product $A1$ (rather than market share) also benefits firm A by limiting the market share that is compatible to the competitor's product $B2$. On the other hand, an increase in market share of product $A1$ improves the competitive positioning of product $A2$, which is compatible only to product $A1$. Hence, focusing on market share for product $A1$ supports $A2$'s market share, which can in turn benefit the competitive position of firm $A1$, as it reduces the externality effect for product $B1$, which is not compatible with product $A2$.

Since these mutually reinforcing effects continue *ad infinitum*, it is difficult to tell the final outcome that A 's opening of market 1 has on prices. Proposition 2b indicates

that under a closed standard, firm A will price lower than firm B in both markets. Only firm A can strategically control the externalities in these closed markets, and we will see below that these lower prices lead to increased market share which benefits A alone.

Under open standards, prices will be identical in both markets.

When firm A chooses to open only one of the markets, say market 1, firm A will undercut the competitor's price in the closed market 2, while it will price higher than the competition in market 1.

Though not customer-centric in terms of functionality, proprietary standards benefit customers in terms of lower prices compared to the open standard case. With closed standards the firm has greater incentive to increase market share in order to capture the beneficial cross-market externalities.

It can easily be shown that, under symmetry, all firms charge prices above their respective cost. We will see that this is not necessarily true in settings where markets or firms are asymmetric (cf. Proposition 6). Under those circumstances, firm A might find it beneficial to price below cost and hence sacrifice profits in one market in order to gain advantages in the complementary market.

We next consider the effect of firm A 's decision to open one or more markets on market share (or sales). Denote resulting market share in market m for firm s 's products as S_{sm}^{open} , S_{sm}^{openm} , $S_{sm}^{open\bar{m}}$, and S_{sm}^{closed} , corresponding to A 's decision to open both markets, market m only, market \bar{m} only, or to close both markets, respectively. Proposition 4 establishes the market share in each market based on A 's decision.

Proposition 3:

- a. $S_{Am}^{openm} > S_{Am}^{closed} > S_{Am}^{open\bar{m}} > S_{Am}^{open}$ and $S_{Bm}^{open} > S_{Bm}^{open\bar{m}} > S_{Bm}^{closed} > S_{Bm}^{openm}$.
- b. $S_{Am}^{open} = S_{Bm}^{open}$, $S_{Am}^{openm} > S_{Bm}^{openm}$, $S_{Am}^{open\bar{m}} > S_{Bm}^{open\bar{m}}$, and $S_{Am}^{closed} > S_{Bm}^{closed}$.

Firm A 's decision to use a closed standard leads to higher market share for A versus an open standard. If firm A opens only one market, market share is relatively high in that market but relatively low in the (closed) complementary market. We found in Proposition 2 that firm A then also increases the price of the product in the open market. Firm A 's product in the open market is more attractive to customers, since it is compatible to all products in the complementary market. It depends on the strength of customer preferences relative to the inter-market externalities whether the utility increase is used by firm A primarily to seek market share or to increase margins. If customers' intra-market preferences are substantial, the market share that firm A stands to gain from low prices is comparably minor, so that firm A can increase the price of the open product more aggressively, translating most of this utility into increased profit margin. On the other hand, for relatively weaker intra-market preferences, a price decrease can have a substantial impact on market share. In this case, firm A passes most of the added utility on to the consumers (i.e., prices are increased only slightly), and firm A accrues benefits in the complementary market.

Firm A 's gain in market share in the open market m comes at firm Bm 's expense, whose relative market share are the mirror image of those of firm A (Proposition 3a). This is also confirmed in Proposition 3b, which contrasts market share for products Am and Bm . Proposition 3b also indicates that if firm A chooses to open

only one market, A 's market share in both markets will still be greater than experienced by the competitors in both markets.

How does a decision to maintain a closed standard affect consumers?

Government interventions to require open markets implicitly assume that closed markets are detrimental to consumers. We saw earlier (Proposition 2a) that open markets might actually be detrimental to consumers in terms of higher prices. In the following proposition, we show that the lower prices associated with proprietary standards more than compensate consumers for the lost utility from the closed markets.

Proposition 4: *Total Consumer Surplus is higher if firm A maintains proprietary standards than if firm A adopts open standards.*

Surprisingly, under proprietary standards, the increased competition induced by firm A 's desire to maximize the beneficial externalities between its markets more than compensates consumers for the reduced accessibility of markets. Counter to the assumptions of well meaning governmental interventions to open markets, consumers can be better off under closed markets.

In our model, closed markets lead to lower profits and prices, but higher market shares. Our model so far has assumed symmetric products. In seeking additional insight, we next relax one of the symmetry assumptions: we allow the reservation prices in one of the markets to differ.

4.2 The Impact of Consumer Preferences for one Product

Throughout this section we assume that the reservation price for product $A1$ differs from that of $B1$ by θ , i.e. the reservation price for product $A1$ is $R_1 + \theta$. We assume no restriction on θ 's sign. Product $B1$ still has a reservation price of R_1 and the two products in market 2 continue to have a reservation price of R_2 .

Proposition 5 compares firm A 's profits versus the decision to open markets, given a preference for Product $A1$.

Proposition 5:

- a. $\pi_A^{open} > \pi_A^{openm} > \pi_A^{closed}$ and $\pi_A^{open1} > \pi_A^{open2} \Leftrightarrow \theta > 0$.
- b. Assume firm A has an advantage in market 1, such that $\theta > 0$. The benefit from opening the two markets (in percentage terms) to firm A decreases in θ and the benefit to firm $B2$ increases in θ . The benefit to firm $B1$ increases in θ , if $\theta < \bar{\theta}$.

This result of Proposition 5a is consistent with the finding for the previously studied case with no advantage (Proposition 1) except that firm A is no longer indifferent between opening market 1 alone and market 2 alone; it would prefer to open the market in which it has an advantage. An advantage in market 1 increases the market share for A in market 1. By opening market 1, firm A benefits from the competitor's market share in market 2, which is not as strongly dominated by firm A . The benefit from opening market 1 is greater than the benefit firm A would obtain from opening market 2, which would give it access to the relatively smaller market share of firm $B1$.

Earlier, under the assumption $\theta = 0$, Proposition 1 stated that firm A 's profits would increase by A 's moving from a closed to an open standard, and Proposition 1 concluded that firms $B1$ and $B2$ both benefited more (in percentage terms) than firm A from A 's decision to open standards. Proposition 5b demonstrates that even if firm A has a more desirable product, firms $B1$ and $B2$ will benefit more than firm A from an open standard. In fact, the more firm A 's product is preferred, the smaller is the percentage improvement from opening standards for firm A , while its competitors' percentage profit improvements grow.⁵

We end this section by noting that if firm A chooses a closed standard, then it may price a product below cost:

Proposition 6: *If (and only if) firm A has a disadvantage in market 1 (i.e. $\theta < 0$) and market 1 is closed (i.e., in scenarios closed or open2), then firm A might price product $A1$ below cost. Independent of θ , firm A never sells product $A2$ below cost.*

Proposition 6 demonstrates that under an open standard, markups are always positive. Under an open standard, firm A is unable to support one of its products by selling its complement at a loss. While reducing the price in one market increases the market share in that specific market, this increase does not provide any advantage in the complementary market, as long as open standards are used. Proposition 6 also indicates

⁵ It can be shown that for any fixed α , the threshold $\bar{\theta}$ (given in the appendix) is strictly increasing in t , indicating that firm A 's advantage likely will be less than the threshold, if intra-market preferences are relatively strong vis-à-vis inter-market externalities.

that unless firm A has a disadvantage in market 1, i.e. unless $\theta < 0$, all markups are positive.

On the other hand, if firm A chooses a proprietary standard for both markets or for market 1 only, there may be sufficient incentive to price product $A1$ below cost in order to support product $A2$. If firm A has the less preferred product in market 1, it might price that product below cost to support the more profitable product in market 2. Firm A never prices below cost in market 2, i.e. in the market where products are equally liked by customers.

4.3 Endogenous Market Size

In the previous sections, we assumed full market coverage in a model with both intra-market competition and cross-market externalities. We found that in such a setting, where total market size is independent of industry prices, a firm that participates in both markets (firm A) in the short term would always be better off by agreeing to open standards, even when its product is more desirable.

We have found closed markets to be suboptimal. However, our assumption of fixed industry sales provides a possible alternative explanation for our findings – if market size varied in industry prices, maintaining a closed standard could actually be more profitable, if lower prices induced a sufficient increase in demand.

In this section, we consider an extension of the base model where each firm's pricing decision affects not only market share, but also market penetration. As before, there are some customers with ideal points between those matched by the two products and we normalize the distance between these two products' locations to one. In order to

capture elasticity of demand with respect to the different prices, we now assume that each firm is no longer at an extreme point of the market. Instead, there are customers with ideal points in the regions beyond the locations of the products of each firm, such that the product from one firm will clearly be preferred by those customers. These customers would not buy a product of the competing standard, and will only buy the preferred supplier's product if it is sufficiently close to the customer's ideal point. Figure 2 illustrates this model for market m .

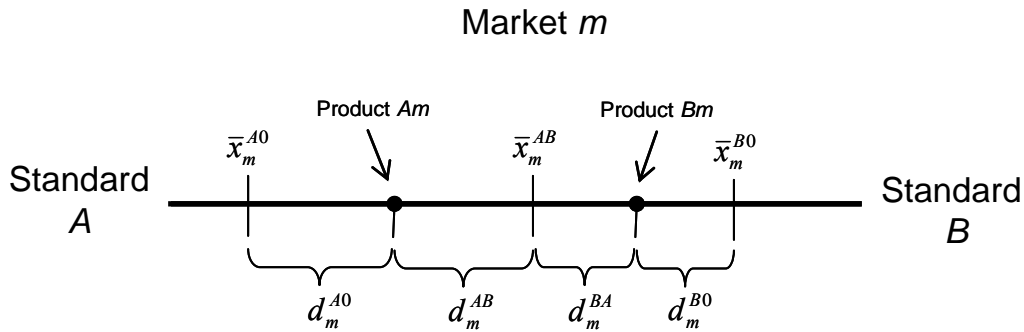


Figure 2: Extension of our Base Model – Capturing Market Penetration (Market m)

To ensure both competition between the firms and positive elasticity of industry demand, we assume that the products' reservation prices are sufficiently high to cover the market between the products' locations but that the markets extend infinitely in each direction such that there will always be some customers with ideal points sufficiently distant from the firm's products that they do not buy. The purpose of this extension of our model is to test whether a firm with presence in both markets (firm A) still finds open standards more profitable than closed standards, given that lower prices under closed standards might increase market penetration. In order to derive analytical insights in this more complex model, in this section we focus on the fully symmetric case, assuming identical costs and reservation prices in both markets.

Since market size now is endogenous, we make a straightforward adjustment to how complementarity effects are modeled. Specifically, we assume that the cross-market externalities are affected by demand rather than market share. (Demand and market share are the same in the base model.)

Using “0” to denote the no-purchase option, d_m^{ij} denotes the distance of the customer \bar{x}_m^{ij} that is indifferent between options i and j , measured from the location of option i (see Figure 2). Sales for the two products in market m can then be determined as $S_{sm} = d_m^{s\bar{s}} + d_m^{s0}$.

The derivation of the different price equilibria is very similar to the base case. Specifically, analogous to Assumption 1 for this case we assume that

$$t > 2\alpha. \quad (\text{Assumption 2})$$

As before, this assumption assures (joint) strict concavity of the optimal price response functions and thereby ensures the existence of a unique price equilibrium. The base case required $t > \alpha$; for stability in this extension with endogenous market size, we need intra-market preferences that are at least twice as strong. The intuition arises from noting that in this model, other things being equal, a price decrease leads to market share gains on *both* sides of the product’s location.

Proposition 7 compares the resulting profits for firm A depending on A ’s decision to open or close standards for the base case extended to consider endogenous market size. Details on the derivation of the equilibria are given in the appendix.

Proposition 7: $\pi_A^{open} > \pi_A^{closed}$

Extending the result in Proposition 1 (for a market of fixed size), we find that firm *A* would profit from adhering to open standards, even in a setting where market size is price dependent.

5. Discussion & Conclusion

Our model suggests that Apple might be pursuing market share, as opposed to singularly focusing on profits. We have seen that, at least in the short term of our single-period model, market share is higher under a closed strategy. But our model does not quantify the benefits of future purchases. What could be the long-term rationale for Apple's resistance to an open standard for music? Consider that in the long run, Apple's customers may be captive. If an Apple customer returns to the MP3 market to buy the next generation hardware, a closed standard will increase the likelihood that the customer remain with Apple – the iPod is the only hardware that would work with the customer's portfolio of music from iTunes. A longer-term strategy may seek to capture benefits that arise with higher market shares over multiple periods.

A closed market strategy has an additional benefit for a dominant player in that it contributes to barriers to entry. More market share (arising from a closed strategy) implies that less market share is available to possible entrants in the complementary markets. An entrant into the music business could only sell to non-iPod customers, while an entrant in the MP3 player market cannot serve iTunes customers. Given Apple's shares in these markets, an entrant will not be able to capture significant externality benefits from the low markets accessible to non-Apple products. In addition, the closed

market strategy results in lower prices, which also contribute as a barrier to entry: there is less money to be made per unit sold into a market with lower prices.

This rationale might provide insights into the difference between the iPod and iTunes results versus those of personal computers and operating systems. Apple chose a closed market for its personal computers and operating system, but its market share never rose much above 12%. When market shares are high, a closed strategy contributes to barriers to entry and captive customers. When market shares are low, the closed market may lose over time with dwindling market shares as customers move to the dominant standard. It is interesting to note that in 2006, Apple has begun to open its personal computer line to the alternate operating system from Microsoft.

The results of our model indicate that Apple's reluctance to open markets for the iPod and iTunes is rational and defensible, but not without tradeoffs. Closed markets garnish higher market share and impede entry of new competitors. Open markets might benefit Apple in the short run, but would benefit and strengthen Apple's current competitors even more than it would benefit Apple. By keeping markets closed Apple maintains and defends its dominant position. And as long as there remains some competition, customers are better off as well. Given a design process that regularly develops attractive new products, closed standards can reinforce Apple's dominance in the market as customers remain with Apple's standard for new generations of iPods.

Apple Computer is a behemoth and smaller players in the MP3 market have been struggling. In 2003, Rio, the pioneer and former leader in the MP3 market was absorbed by D&M Holdings after filing for Chapter 11 bankruptcy protection. On the other hand, key players, such as Microsoft, Dell, and Amazon have interest in the market. A recent

Business Week article states that “after attacking the Sony PlayStation with the Xbox, insiders are waiting for Microsoft to unveil its XPod – ‘an iPod killer’” (Greene 2006). The Wall Street Journal reports on Amazon’s plan to enter the music and player market to lesson their dependence on books and CDs (Smith and Mangalindan 2006). Keeping Apple’s markets closed makes it more difficult for these (and other) players to enter the market.

In this paper we have shown that Apple’s strategy to maintain a proprietary standard for its iPod and iTunes products might be consistent with a profitable long-term objective that gains significant market share and maintains it by providing customers an incentive to stay with the standard for future purchases.

Our model is a stylized abstraction of a specific ownership structure. Some additional insights might arise from the relaxation of our various symmetry constraints. In addition, it might be fruitful to use our modeling framework to analyze alternative ownership structures. For example, in addition to having firm A own products in both markets, there could be a single firm B that competes with products in both markets. Alternatively, there could be four independent firms each with a product in one of the two markets.

Competition in complementary markets arises in multiple settings and various incarnations. In the 1980s, JVC’s VHS video ultimately prevailed over Sony’s Betamax. The stage is now set for a possible reprise featuring DVD recordings in high definition. Sony is determined to prevail this time with its Blu-ray technology in competition with the rival technology from Toshiba Corp. The HD DVD battle is a variant of our model in that the markets are undeveloped. The battle is ensuing before there is *any* market share.

There is tremendous concern that two closed standards will result in market confusion and significantly impede customer acceptance of the technology. Thus, Sony and Toshiba both seek an open standard – but each player seeks for theirs to be the standard. Sony will participate in both markets: in addition to selling DVD players, Sony owns much of the content (i.e., movies) sold in the complementary market for DVD players.

Other examples of competition in complementary markets include operating systems and office applications (e.g., Microsoft Windows and Office), Internet providers and Voice Over Internet Phone (VOIP) service (Time Warner and others), printers and ink cartridges, and razors and blades. We have seen that when there is competition in complementary markets, a firm that participates in both markets can gain market share and increase barriers to entry by seeking closure.

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Appendix

Derivation of the Equilibria

We consider 4 cases, depending on firm A 's decision to open both markets, market 1 only, market 2 only, or to close both markets, respectively.

Firm A 's decision affects which products in the two markets are compatible (e.g., under the closed scenario, product $A1$ can only be used with product $A2$, whereas it can be used with either product of type 2 in the open1 scenario or the open scenario) and it thus directly affects the magnitude of the cross-market externalities. Specifically, adjusted reservation prices are determined as follows:

<u>Scenario "closed"</u>	<u>Scenario "open"</u>
$\hat{R}_{A1} = R_1 + \theta + \alpha S_{A2}$	$\hat{R}_{A1} = R_1 + \theta + \alpha$
$\hat{R}_{A2} = R_2 + \alpha S_{A1}$	$\hat{R}_{A2} = R_2 + \alpha$
$\hat{R}_{B1} = R_1 + \alpha S_{B2}$	$\hat{R}_{B1} = R_1 + \alpha$
$\hat{R}_{B2} = R_2 + \alpha S_{B1}$	$\hat{R}_{B2} = R_2 + \alpha$
<u>Scenario "open1"</u>	<u>Scenario "open2"</u>
$\hat{R}_{A1} = R_1 + \theta + \alpha$	$\hat{R}_{A1} = R_1 + \theta + \alpha S_{A2}$
$\hat{R}_{A2} = R_2 + \alpha S_{A1}$	$\hat{R}_{A2} = R_2 + \alpha$
$\hat{R}_{B1} = R_1 + \alpha S_{B2}$	$\hat{R}_{B1} = R_1 + \alpha$
$\hat{R}_{B2} = R_2 + \alpha$	$\hat{R}_{B2} = R_2 + \alpha S_{B1}$

Given our ownership structure, in each of these cases, firm A maximizes joint profits $\pi_A = \pi_{A1} + \pi_{A2}$, whereas firms $B1$ and $B2$ independently maximize their individual profits π_{B1} and π_{B2} , respectively. It can be easily shown that, under Assumptions 1 and 2, the profit functions are strictly (jointly) concave in the respective prices, and simultaneously solving the set of necessary and sufficient first-order

conditions $\left\{ \frac{\partial \pi_A}{\partial P_{A1}} = 0, \frac{\partial \pi_A}{\partial P_{A2}} = 0, \frac{\partial \pi_{B1}}{\partial P_{B1}} = 0, \frac{\partial \pi_{B2}}{\partial P_{B2}} = 0 \right\}$ yields the following equilibrium

prices:

$$P_{A1}^{closed} = C_1 + \frac{9t^3 - 6t^2\alpha - 5t\alpha^2 + 2\alpha^3 + 3t^2\theta}{(3t - \alpha)(3t + \alpha)}$$

$$P_{A2}^{closed} = C_2 + \frac{9t^3 - 6t^2\alpha - 5t\alpha^2 + 2\alpha^3 - t\alpha\theta}{(3t - \alpha)(3t + \alpha)}$$

$$P_{B1}^{closed} = C_1 + \frac{9t^3 - 3t^2\alpha - 9t\alpha^2 + 3\alpha^3 - 3t^2\theta + \alpha^2\theta}{(3t - \alpha)(3t + \alpha)}$$

$$P_{B2}^{closed} = C_2 + \frac{9t^3 - 3t^2\alpha - 9t\alpha^2 + 3\alpha^3 - 2t\alpha\theta}{(3t - \alpha)(3t + \alpha)}$$

$$P_{A1}^{open} = C_1 + t + \frac{\theta}{3}$$

$$P_{A2}^{open} = C_2 + t$$

$$P_{B1}^{open} = C_1 + t - \frac{\theta}{3}$$

$$P_{B2}^{open} = C_2 + t$$

$$P_{A1}^{open1} = C_1 + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3 + 12t^2\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{A2}^{open1} = C_2 + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3 - 2t\alpha\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{B1}^{open1} = C_1 + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3 - 12t^2\theta + \alpha^2\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{B2}^{open1} = C_2 + \frac{36t^3 - 11t\alpha^2 + \alpha^3 - 4t\alpha\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{A1}^{open2} = C_1 + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3 + 12t^2\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{A2}^{open2} = C_2 + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3 - 2t\alpha\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{B1}^{open2} = C_1 + \frac{36t^3 - 11t^2\alpha + \alpha^3 - 12t^2\theta + \alpha^2\theta}{(6t - \alpha)(6t + \alpha)}$$

$$P_{B2}^{open2} = C_2 + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3 - 4t\alpha\theta}{(6t - \alpha)(6t + \alpha)}$$

The resulting equilibrium profits and demands for each case can be obtained by substituting the corresponding prices into the respective functions (see equation 1 and following discussion).

Parametric Conditions for Interior Equilibria

Our focus on interior solutions implies the following limitations on the difference in reservation prices:

$$0 < S_{sm}^{open} < 1 \Leftrightarrow -3t < \theta < 3t \quad (L1)$$

$$0 < S_{s1}^{open1} < 1 \Leftrightarrow -\frac{(2t-\alpha)(36t^3+30t^2\alpha+2t\alpha^2-\alpha^3)}{2t(12t^2-\alpha^2)} < \theta < \frac{(2t-\alpha)(18t^2+3t\alpha-2\alpha^2)}{12t^2-\alpha^2} \quad (L2)$$

$$0 < S_{s2}^{open1} < 1 \Leftrightarrow -\frac{(2t-\alpha)(36t^3+18t^2\alpha-\alpha^3)}{8t^2\alpha} < \theta < \frac{(2t-\alpha)(18t^2+9t\alpha-\alpha^2)}{4t\alpha} \quad (L3)$$

$$0 < S_{s1}^{open2} < 1 \Leftrightarrow -\frac{(2t-\alpha)(36t^3+18t^2\alpha-\alpha^3)}{2t(12t^2-\alpha^2)} < \theta < \frac{(2t-\alpha)(18t^2+9t\alpha-\alpha^2)}{12t^2-\alpha^2} \quad (L4)$$

$$0 < S_{s2}^{open2} < 1 \Leftrightarrow -\frac{(2t-\alpha)(36t^3+30t^2\alpha+2t\alpha^2-\alpha^3)}{8t^2\alpha} < \theta < \frac{(2t-\alpha)(18t^2+3t\alpha-2\alpha^2)}{4t\alpha} \quad (L5)$$

$$0 < S_{s1}^{closed} < 1 \Leftrightarrow -\frac{3(t-\alpha)(t+\alpha)(3t-\alpha)}{3t^2-\alpha^2} < \theta < \frac{(t-\alpha)(t+\alpha)(3t-\alpha)(3t+2\alpha)}{t(3t^2-\alpha^2)} \quad (L6)$$

$$0 < S_{s2}^{closed} < 1 \Leftrightarrow -\frac{(t-\alpha)(t+\alpha)(3t-\alpha)(3t+2\alpha)}{2t^2\alpha} < \theta < \frac{3(t-\alpha)(t+\alpha)(3t-\alpha)}{2t\alpha} \quad (L7)$$

Proof of Proposition 1: Part a) For $\theta = 0$, simplification gives $\pi_A^{open} = t$,

$$\pi_A^{openm} = \frac{2592t^6 + 864t^5\alpha - 792t^4\alpha^2 - 240t^3\alpha^3 + 64t^2\alpha^4 + 12t\alpha^5 - 2\alpha^6}{(2t+\alpha)(36t^2-\alpha^2)^2},$$

$$\pi_A^{closed} = \frac{(t-\alpha)(3t+2\alpha)^2}{(3t+\alpha)^2}, \quad \pi_{Bm}^{open} = \frac{t}{2}, \quad \pi_{Bm}^{openm} = \frac{2t(2t-\alpha)(18t^2+9t\alpha-\alpha^2)^2}{(2t+\alpha)(36t^2-\alpha^2)^2},$$

$$\pi_{Bm}^{openm} = \frac{2t(2t-\alpha)(18t^2+3t\alpha-2\alpha^2)^2}{(2t+\alpha)(36t^2-\alpha^2)^2}, \quad \text{and} \quad \pi_{Bm}^{closed} = \frac{9t(t-\alpha)(t+\alpha)}{2(3t+\alpha)^2}.$$

Recall that $t > \alpha$ by Assumption 1.

$$\pi_A^{open} > \pi_A^{openm} \Leftrightarrow t > \frac{2592t^6 + 864t^5\alpha - 792t^4\alpha^2 - 240t^3\alpha^3 + 64t^2\alpha^4 + 12t\alpha^5 - 2\alpha^6}{(2t+\alpha)(26t^2-\alpha^2)^2}$$

$$\Leftrightarrow 432t^5 + 648t^4\alpha + 95t^3\alpha^2 + 62t^2\alpha^2(t-\alpha) + 11t\alpha^2(t^2-\alpha^2) + 2\alpha^5 > 0.$$

$$\pi_A^{openm} > \pi_A^{closed}$$

$$\Leftrightarrow \frac{2592t^6 + 864t^5\alpha - 792t^4\alpha^2 - 240t^3\alpha^3 + 64t^2\alpha^4 + 12t\alpha^5 - 2\alpha^6}{(2t+\alpha)(26t^2-\alpha^2)^2} > \frac{(t-\alpha)(3t+2\alpha)^2}{(3t+\alpha)^2}$$

$$\Leftrightarrow 3888t^7 + 18792t^6\alpha + 15768t^5\alpha^2 + 1502t^4\alpha^3 + 915t^3\alpha^3(t - \alpha) + 157t^2\alpha^3(t^2 - \alpha^2) + 16t\alpha^6 + 2\alpha^7 > 0.$$

$$\pi_{Bm}^{open} > \pi_{Bm}^{open\bar{m}} \Leftrightarrow \frac{t}{2} > \frac{2t(2t - \alpha)(18t^2 + 9t\alpha - \alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2}$$

$$\Leftrightarrow 792t^3 + 174t^2\alpha + 78t\alpha(t - \alpha) + 5\alpha^3 > 0.$$

$$\pi_{Bm}^{openm} > \pi_{Bm}^{closed} \Leftrightarrow \frac{2t(2t - \alpha)(18t^2 + 3t\alpha - 2\alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2} > \frac{9t(t - \alpha)(t + \alpha)}{2(3t + \alpha)^2}$$

$$\Leftrightarrow 16200t^5 + 6902t^4\alpha + 594t^3\alpha(t - \alpha) + 165t^2\alpha(t^2 - \alpha^2) + 2t\alpha^4 + 7\alpha(t^4 - \alpha^4) > 0.$$

$$\pi_{Bm}^{open\bar{m}} > \pi_{Bm}^{openm} \Leftrightarrow \frac{2t(2t - \alpha)(18t^2 + 9t\alpha - \alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2} > \frac{2t(2t - \alpha)(18t^2 + 3t\alpha - 2\alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2}$$

$$\Leftrightarrow (18t^2 + 9t\alpha - \alpha^2)^2 > (18t^2 + 3t\alpha - 2\alpha^2)^2 \Leftrightarrow 6t + \alpha > 0.$$

Part b) $\pi_A^{open} = t = 2\frac{t}{2} = 2\pi_{Bm}^{open}.$

$$\pi_A^{closed} > \pi_{B1}^{closed} + \pi_{B2}^{closed} \Leftrightarrow \frac{(t - \alpha)(3t + 2\alpha)^2}{(3t + \alpha)^2} > 2\frac{9t(t - \alpha)(t + \alpha)}{2(3t + \alpha)^2} \Leftrightarrow$$

$$(3t + 2\alpha)^2 > 9t(t + \alpha) \Leftrightarrow 3t + 4\alpha > 0.$$

$$\pi_A^{openm} > \pi_{B1}^{openm} + \pi_{B2}^{openm}$$

$$\Leftrightarrow \frac{2592t^6 + 864t^5\alpha - 792t^4\alpha^2 - 240t^3\alpha^3 + 64t^2\alpha^4 + 12t\alpha^5 - 2\alpha^6}{(2t + \alpha)(36t^2 - \alpha^2)^2}$$

$$> \frac{2t(2t - \alpha)(18t^2 + 9t\alpha - \alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2} + \frac{2t(2t - \alpha)(18t^2 + 3t\alpha - 2\alpha^2)^2}{(2t + \alpha)(36t^2 - \alpha^2)^2}$$

$$\Leftrightarrow 129t^5 + 72t^4\alpha + 78t^3(t^2 - \alpha^2) + 8t^2(t^3 - \alpha^3) + 11t\alpha^4 + (t^5 - \alpha^5) > 0.$$

Part c) $\frac{\pi_A^{open} - \pi_A^{closed}}{\pi_A^{closed}} < \frac{\pi_{Bm}^{open} - \pi_{Bm}^{closed}}{\pi_{Bm}^{closed}} \Leftrightarrow t \frac{(3t + \alpha)^2}{(t - \alpha)(3t + 2\alpha)^2} - 1 < \frac{t}{2} \frac{2(3t + \alpha)^2}{9t(t - \alpha)(t + \alpha)} - 1$

$$\Leftrightarrow (3t + 2\alpha)^2 > 9t(t + \alpha) \Leftrightarrow 3t + 4\alpha > 0. \blacksquare$$

Proof of Proposition 2: Part a) For $\theta = 0$, simplification gives $P_{Am}^{open} = P_{Bm}^{open} = C_m + t$,

$$P_{Am}^{openm} = C_m + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)}, \quad P_{Am}^{open\bar{m}} = C_m + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)},$$

$$P_{Am}^{closed} = C_m + \frac{(t - \alpha)(3t + 2\alpha)}{3t + \alpha}, \quad P_{Bm}^{open\bar{m}} = C_m + \frac{36t^3 - 11t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)},$$

$$P_{Bm}^{openm} = C_m + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3}{(6t - \alpha)(6t + \alpha)}, \quad \text{and } P_{Bm}^{closed} = C_m + \frac{3(t - \alpha)(t + \alpha)}{3t + \alpha}. \text{ Recall that}$$

$t > \alpha$ by Assumption 1. Then

$$P_{Am}^{open} > P_{Am}^{openm} \Leftrightarrow C_m + t > C_m + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} \Leftrightarrow 6t^2 + 2t\alpha + 2\alpha(t - \alpha) > 0.$$

$$P_{Am}^{openm} > P_{Am}^{open\bar{m}} \Leftrightarrow C_m + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} > C_m + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)}$$

$$\Leftrightarrow 6t + \alpha > 0.$$

$$P_{Am}^{open\bar{m}} > P_{Am}^{closed} \Leftrightarrow C_m + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} > C_m + \frac{(t - \alpha)(3t + 2\alpha)}{3t + \alpha}$$

$$\Leftrightarrow 18t^3 + 34t^2\alpha + 4t\alpha(t - \alpha) + \alpha(t^2 - \alpha^2) > 0.$$

$$P_{Bm}^{open} > P_{Bm}^{open\bar{m}} \Leftrightarrow C_m + t > C_m + \frac{36t^3 - 11t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} \Leftrightarrow 10t > \alpha.$$

$$P_{Bm}^{open\bar{m}} > P_{Bm}^{openm} \Leftrightarrow C_m + \frac{36t^3 - 11t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} > C_m + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3}{(6t - \alpha)(6t + \alpha)}$$

$$\Leftrightarrow \frac{\alpha(2t - \alpha)}{(6t - \alpha)} > 0.$$

$$P_{Bm}^{openm} > P_{Bm}^{closed} \Leftrightarrow C_m + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3}{(6t - \alpha)(6t + \alpha)} > C_m + \frac{3(t - \alpha)(t + \alpha)}{3t + \alpha}$$

$$\Leftrightarrow 76t^2 + t(t - \alpha) + (t^2 - \alpha^2) > 0.$$

Part b)

$$P_{Am}^{closed} < P_{Bm}^{closed} \Leftrightarrow C_m + \frac{(t - \alpha)(3t + 2\alpha)}{3t + \alpha} < C_m + \frac{3(t - \alpha)(t + \alpha)}{3t + \alpha} \Leftrightarrow (3t + 2\alpha) < 3(t + \alpha).$$

$$P_{Am}^{open\bar{m}} < P_{Bm}^{open\bar{m}} \Leftrightarrow C_m + \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} < C_m + \frac{36t^3 - 11t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} \quad 18t > 5\alpha.$$

$$P_{Am}^{openm} > P_{Bm}^{openm} \Leftrightarrow C_m + \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{(6t - \alpha)(6t + \alpha)} > C_m + \frac{36t^3 - 12t^2\alpha - 7t\alpha^2 + 2\alpha^3}{(6t - \alpha)(6t + \alpha)}$$

$$\Leftrightarrow 6t^2\alpha + 2t\alpha^2 + \alpha^2(t - \alpha) > 0. \quad P_{Am}^{open} = P_{Bm}^{open} = C_m + t. \blacksquare$$

Proof of Proposition 3: Part a) For the symmetric case with $\theta = 0$, simplification gives

$$S_{Am}^{openm} = \frac{36t^3 + 30t^2\alpha + 2t\alpha^2 - \alpha^3}{(6t - \alpha)(6t + \alpha)(2t + \alpha)}, \quad S_{Am}^{closed} = \frac{3t + 2\alpha}{2(3t + \alpha)}, \quad S_{Am}^{open} = \frac{1}{2}, \quad \text{and}$$

$$S_{Am}^{open\bar{m}} = \frac{36t^3 + 18t^2\alpha - \alpha^3}{(6t - \alpha)(6t + \alpha)(2t + \alpha)}. \quad \text{Recall that } t > \alpha \text{ by Assumption 1. Then}$$

$$S_{Am}^{openm} > S_{Am}^{closed} \Leftrightarrow \frac{36t^3 + 30t^2\alpha + 2t\alpha^2 - \alpha^3}{(6t - \alpha)(6t + \alpha)(2t + \alpha)} > \frac{3t + 2\alpha}{2(3t + \alpha)} \Leftrightarrow 6t + 5\alpha > 0.$$

$$S_{Am}^{closed} > S_{Am}^{open\bar{m}} \Leftrightarrow \frac{3t+2\alpha}{2(3t+\alpha)} > \frac{36t^3+18t^2\alpha-\alpha^3}{(6t-\alpha)(6t+\alpha)(2t+\alpha)} \Leftrightarrow 72t^2+20t\alpha+\alpha(t-\alpha) > 0.$$

$$S_{Am}^{open\bar{m}} > S_{Am}^{open} \Leftrightarrow \frac{36t^3+18t^2\alpha-\alpha^3}{(6t-\alpha)(6t+\alpha)(2t+\alpha)} > \frac{1}{2} \Leftrightarrow 2t-\alpha > 0.$$

The corresponding results for the competitor's market shares follow directly from the assumptions of full market coverage and fixed market size.

Part b) Since $S_{Am}^{open} = 1/2$, this follows from the assumptions of full market coverage and fixed market size and from the ordering given in part a). ■

Proof of Proposition 4: On average, a consumer who purchases product sm in scenario

k derives a surplus of $CS_{sm}^k = \hat{R}_{sm}^k - P_{sm}^k - t \frac{S_{sm}^k}{2}$. Hence, total consumer surplus in

scenario k is $CS^k = \sum_{s,m} S_{sm}^k CS_{sm}^k$. Substitution of the equilibrium prices and subsequent

$$\text{simplification gives } CS^{closed} = R_1 + R_2 - C_1 - C_2 - \frac{45t^3 - 6t^2\alpha + 44t\alpha^2 + 12\alpha^3}{2(3t+\alpha)^2},$$

$$CS^{open} = R_1 + R_2 - C_1 - C_2 - \frac{5t-4\alpha}{2}, \text{ and } CS^{closed} - CS^{open} = \frac{\alpha^2(25t+8\alpha)}{2(3t+\alpha)^2} > 0. \blacksquare$$

Proof of Proposition 5: Part a) Simplification gives $\pi_A^{open} = t + \frac{\theta}{3} + \frac{\theta^2}{18t}$,

$$\begin{aligned} \pi_A^{open1} = & \left(2(2t-\alpha)(1296t^6 + 432t^5\alpha - 396t^4\alpha^2 - 120t^3\alpha^3 + 32t^2\alpha^4 + 6t\alpha^5 - \alpha^6) \right. \\ & \left. + 4t(2t-\alpha)(216t^4 + 144t^3\alpha - 6t^2\alpha^2 - 6t\alpha^3 + \alpha^4)\theta + 8t^3(36t^2 - 5\alpha^2)\theta^2 \right) \\ & / \left((2t+\alpha)(2t-\alpha)(6t+\alpha)^2(6t-\alpha)^2 \right), \end{aligned}$$

$$\begin{aligned} \pi_A^{open2} = & \left(2(2t-\alpha)(1296t^6 + 432t^5\alpha - 396t^4\alpha^2 - 120t^3\alpha^3 + 32t^2\alpha^4 + 6t\alpha^5 - \alpha^6) \right. \\ & \left. + 4t(2t-\alpha)(216t^4 + 72t^3\alpha - 30t^2\alpha^2 - 8t\alpha^3 + \alpha^4)\theta + 8t^3(36t^2 - 5\alpha^2)\theta^2 \right) \\ & / \left((2t+\alpha)(2t-\alpha)(6t+\alpha)^2(6t-\alpha)^2 \right), \text{ and} \end{aligned}$$

$$\pi_A^{closed} = \frac{(t-\alpha)(3t+2\alpha)^2}{(3t+\alpha)^2} + \frac{t(3t+2\alpha)}{(3t+\alpha)^2}\theta + \frac{t^3(9t^2-5\alpha^2)}{2(t+\alpha)(t-\alpha)(9t^2-\alpha^2)^2}\theta^2.$$

Recall that $t > \alpha$ by Assumption 1. Since $\pi_A^{open1} - \pi_A^{open2} > 0 \Leftrightarrow \frac{8t^2\alpha\theta}{(2t+\alpha)(6t-\alpha)^2} > 0$

$\Leftrightarrow \theta > 0$, we can limit the proof of $\pi_A^{open} > \pi_A^{openm}$ to showing that $\pi_A^{open} > \pi_A^{open1}$ for positive values of θ , and that $\pi_A^{open} > \pi_A^{open2}$ for negative values of θ .

Comparing the coefficients of the quadratic elements, we have

$$\frac{8t^3(36t^2-5\alpha^2)}{(2t+\alpha)(2t-\alpha)(6t+\alpha)^2(6t-\alpha)^2} > \frac{1}{18t} \Leftrightarrow 788t^4 + 76t^2(t^2-\alpha^2) + \alpha^4 > 0, \text{ so the}$$

difference function $\pi_A^{open} - \pi_A^{open1}$ is a concave quadratic function in θ . Hence it is sufficient to show it is positive at the end points of the feasible region (here we use L1).

At $\theta = 0$, $\pi_A^{open} > \pi_A^{open1}$ follows from Proposition 1.

At $\theta = 3t$, simplification gives $\pi_A^{open} - \pi_A^{open1} > 0$

$$\Leftrightarrow 408t^5 + 480t^4(t - \alpha) + 548t^3(t^2 - \alpha^2) + 104t^2\alpha^3 + 27t\alpha^4 + 4(t^5 - \alpha^5) > 0$$

Proof of $\pi_A^{open} > \pi_A^{open2}$: Since π_A^{open2} has the same coefficient at θ^2 as π_A^{open1} , the difference function $\pi_A^{open} - \pi_A^{open2}$ also is a concave quadratic function in θ .

At $\theta = 0$, $\pi_A^{open} > \pi_A^{open2}$ follows from Proposition 1.

At $\theta = -3t$, simplification gives $\pi_A^{open} - \pi_A^{open2} > 0$

$$\Leftrightarrow 152t^5 + 288t^4(t - \alpha) + 420t^3(t^2 - \alpha^2) + 56t^2\alpha^3 + 31t\alpha^4 + 4(t^5 - \alpha^5) > 0.$$

Since $\pi_A^{open1} - \pi_A^{open2} > 0 \Leftrightarrow \theta > 0$, we can limit the proof of $\pi_A^{openm} > \pi_A^{closed}$ to showing that $\pi_A^{open1} > \pi_A^{closed}$ for negative values of θ , and that $\pi_A^{open2} > \pi_A^{closed}$ for positive values of θ .

$$\text{Since } \frac{t^3(9t^2 - 5\alpha^2)}{2(t + \alpha)(t - \alpha)(9t^2 - \alpha^2)^2} > \frac{8t^3(36t^2 - 5\alpha^2)}{(2t + \alpha)(2t - \alpha)(6t + \alpha)^2(6t - \alpha)^2}$$

$\Leftrightarrow 4331t^6 + 3420t^4(t^2 - \alpha^2) + 569t^2\alpha^4 + 25(t^6 - \alpha^6) > 0$, the difference function $\pi_A^{open1} - \pi_A^{closed}$ is a concave quadratic function in θ . Hence it is sufficient to show that it is positive at the end points of the feasible region (here we use L6).

At $\theta = 0$, $\pi_A^{open1} > \pi_A^{closed}$ follows from Proposition 1.

At $\theta = -\frac{3(t - \alpha)(t + \alpha)(3t - \alpha)}{3t^2 - \alpha^2}$, simplification gives $\pi_A^{open1} - \pi_A^{closed} > 0$

$$\begin{aligned} \Leftrightarrow & 163620t^{11} + 23112t^{10}\alpha + 139644t^9(t^2 - \alpha^2) + 373464t^8\alpha(t^2 - \alpha^2) \\ & + 27009t^7\alpha^4 + 105138t^6\alpha^5 + 1674t^5\alpha^5(t - \alpha) + 15408t^4\alpha^5(t^2 - \alpha^2) \\ & + 401t^3\alpha^8 + 464t^2\alpha^9 + 36t\alpha^9(t - \alpha) + 4\alpha^9(t^2 - \alpha^2) > 0 \end{aligned} \quad (C1)$$

Since π_A^{open2} has the same coefficient at θ^2 as π_A^{open1} , the difference function $\pi_A^{open2} - \pi_A^{closed}$ also is a concave quadratic function in θ . Hence it is sufficient to show that it is positive at the end points of the feasible region.

At $\theta = 0$, $\pi_A^{open2} > \pi_A^{closed}$ follows from Proposition 1.

At $\theta = \frac{(t - \alpha)(t + \alpha)(3t - \alpha)(3t + 2\alpha)}{t(3t^2 - \alpha^2)}$, simplification of $\pi_A^{open2} - \pi_A^{closed} > 0$ gives

condition (C1).

Part b): Profits of the competing firm can be simplified to $\pi_{B1}^{open} = \frac{(3t - \theta)^2}{18t}$, $\pi_{B2}^{open} = \frac{t}{2}$,

$$\pi_{B1}^{closed} = \frac{t(9t^3 - 3t^2\alpha - 9t\alpha^2 + 3\alpha^3 - \theta(3t^2 - \alpha^2))^2}{2(t + \alpha)(t - \alpha)(3t - \alpha)^2(3t + \alpha)^2}, \text{ and}$$

$$\pi_{B2}^{closed} = \frac{t(9t^3 - 3t^2\alpha - 9t\alpha^2 + 3\alpha^3 - 2t\alpha\theta)^2}{2(t + \alpha)(t - \alpha)(3t - \alpha)^2(3t + \alpha)^2}. \text{ Recall that } t > \alpha \text{ by Assumption 1.}$$

Then $\frac{\partial}{\partial \theta} \left(\frac{\pi_{B2}^{open} - \pi_{B2}^{closed}}{\pi_{B2}^{closed}} \right) > 0 \Leftrightarrow \theta < \frac{3(t - \alpha)(t + \alpha)(3t - \alpha)}{2t\alpha}$ is true by (L7).

$$\begin{aligned} \frac{\partial}{\partial \theta} \left(\frac{\pi_A^{open} - \pi_A^{closed}}{\pi_A^{closed}} \right) < 0 \Leftrightarrow & 6t(t - \alpha)(t + \alpha)(t + 2\alpha)(3t + 2\alpha)(3t - \alpha)^2 \\ & + 2\theta(5t^6 + 22t^4(t^2 - \alpha^2) + 20t^4\alpha(t - \alpha) + 97t^3\alpha(t^2 - \alpha^2) + 19t^2\alpha^4 + 12t\alpha^5 + 4\alpha^5(t - \alpha)) \\ & + t\alpha\theta^2(10t^3 + 2t^2(t - \alpha) + 9t(t^2 - \alpha^2) + 2\alpha^3) \text{ is true for } \theta > 0. \end{aligned}$$

Using (L1), we have $\frac{\partial}{\partial \theta} \left(\frac{\pi_{B1}^{open} - \pi_{B1}^{closed}}{\pi_{B1}^{closed}} \right) > 0 \Leftrightarrow \theta < \bar{\theta} = \frac{3(t - \alpha)(t + \alpha)(3t - \alpha)}{3t^2 - \alpha^2}$. We

provide two numeric examples to show that there can be instances with $\theta > \bar{\theta}$ and with $\theta < \bar{\theta}$. With $t = 1$, $\alpha = 1/4$, we have $\theta > \bar{\theta}$ for $\theta = 87/32$ and $\theta < \bar{\theta}$ for $\theta = -21/8$. Both of these numeric examples are within the feasible region defined by (L1)-(L7), which implies that the threshold can lie within the feasible region. ■

Proof of Proposition 6: Recall that $t > \alpha$ by Assumption 1.

$$P_{A1}^{open} > C_1 \Leftrightarrow \theta > -3t, \text{ which is true by (L1).}$$

$$P_{A2}^{open} > C_2 \Leftrightarrow t > 0$$

$$P_{A1}^{open1} > C_1 \Leftrightarrow \theta > -\frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{12t^2}, \text{ which is true by (L6), since}$$

$$-\frac{3(t - \alpha)(t + \alpha)(3t - \alpha)}{3t^2 - \alpha^2} > -\frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{12t^2}$$

$$\Leftrightarrow 18t^4 + 33t^3\alpha + 27t^2\alpha(t - \alpha) + 3t\alpha^3 + \alpha^3(t - \alpha) > 0.$$

$$P_{A2}^{open1} > C_2 \Leftrightarrow \theta < \frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{2t\alpha} \text{ which is true by (L2), since}$$

$$\frac{36t^3 - 18t^2\alpha - 6t\alpha^2 + \alpha^3}{2t\alpha} > \frac{(2t - \alpha)(18t^2 + 3t\alpha - 2\alpha^2)}{12t^2 - \alpha^2}$$

$$\Leftrightarrow 2t^3 + 8t^2(t - \alpha) + 2t(t^2 - \alpha^2) + \alpha^3 > 0.$$

$$P_{A2}^{closed} > C_2 \Leftrightarrow \theta < \frac{(t - \alpha)(3t - \alpha)(3t + 2\alpha)}{t\alpha} \text{ which is true by (L6), since}$$

$$\frac{(t - \alpha)(3t - \alpha)(3t + 2\alpha)}{t\alpha} > \frac{(t - \alpha)(t + \alpha)(3t - \alpha)(3t + 2\alpha)}{t(3t^2 - \alpha^2)}$$

$$\Leftrightarrow t(t - \alpha) + 2(t^2 - \alpha^2) > 0.$$

$P_{A2}^{open2} > C_2 \Leftrightarrow \theta < \frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{2t\alpha}$ which is true by (L5), since

$$\frac{36t^3 - 6t^2\alpha - 4t\alpha^2 + \alpha^3}{2t\alpha} > \frac{(2t - \alpha)(18t^2 + 3t\alpha - 2\alpha^2)}{4t\alpha}$$

$$\Leftrightarrow (6t - \alpha)(6t + \alpha) > 0.$$

If it has an advantage in market 1 ($\theta > 0$), firm A always prices above cost. For $\theta > 0$:

$P_{A1}^{closed} > C_1 \Leftrightarrow \theta > -\frac{(t - \alpha)(3t - \alpha)(3t + 2\alpha)}{3t^2}$, and this threshold is negative.

$P_{A1}^{open2} > C_1 \Leftrightarrow \theta > -\frac{12t^3 + 18t^2(t - \alpha) + 6t(t^2 - \alpha^2) + \alpha^3}{12t^2}$, and this threshold is negative.

When firm A has a disadvantage in market 1 ($\theta < 0$) and market 1 is closed, firm A might price below cost in market 1. We prove this statement by example: We have $P_{A1}^{closed} < C_1$ for $t=1$, $\alpha=1/4$, and $\theta=-21/8$ and $P_{A1}^{open2} < C_1$ for $t=1$, $\alpha=5/16$, and $\theta=-639/256$. It can easily be verified that these numerical examples satisfy the conditions for interior solutions (L1)-(L7).■

Proof of Proposition 7: We first sketch the derivation of the equilibria under the “open” and the “closed” scenarios for the case where total market demand is price elastic. We limit this derivation to the case with $\theta = 0$. The proof of the proposition (using these equilibrium profits) is contained in the second part.

Derivation of the equilibrium profits

For the “closed” scenario, the customer that is indifferent between product sm and the no-purchase option is determined through the equality $U_{sm}(d_m^{s0}) = R + \alpha S_{sm} - P_{sm} - td_m^{s0} = 0$,

so each product captures a market segment of $d_m^{s0} = \frac{R + \alpha S_{sm} - P_{sm}}{t}$ on the outside.

The market on the inside is split at the location of customer that is indifferent between the two products. This customer is determined by the equality

$$U_{Am}(d_m^{AB}) = U_{Bm}(d_m^{BA} = 1 - d_m^{AB})$$

$$\Leftrightarrow R + \alpha S_{A\bar{m}} - P_{Am} - td_m^{AB} = R + \alpha S_{B\bar{m}} - P_{Bm} - t(1 - d_m^{AB}), \text{ so}$$

$$d_m^{AB} = \frac{1}{2} + \frac{\alpha(S_{A\bar{m}} - S_{B\bar{m}}) - (P_{Am} - P_{Bm})}{2t}.$$

Similarly, for the “open” scenario, each product captures a market segment

$d_m^{s0} = \frac{R - P_{sm} + \alpha(S_{s\bar{m}} + S_{\bar{s}m})}{t}$ on the outside and the market on the inside is split at the

location of the indifferent customer $d_m^{AB} = \frac{1}{2} - \frac{(P_{Am} - P_{Bm})}{2t}$.

In both cases sales of the individual products can be derived by simultaneously solving the equation $S_{sm} = d_m^{ss} + d_m^{s0}$ for all four products. Profits associated with product sm equal $\pi_{sm} = (P_{sm} - C)S_{sm}$.

As before, firm A maximizes joint profits $\pi_A = \pi_{A1} + \pi_{A2}$, whereas firms $B1$ and $B2$ independently maximize their individual profits π_{B1} and π_{B2} , respectively.

Simultaneously solving the set of necessary and sufficient first-order conditions

$$\left\{ \frac{\partial \pi_A}{\partial P_{A1}} = 0, \frac{\partial \pi_A}{\partial P_{A2}} = 0, \frac{\partial \pi_{B1}}{\partial P_{B1}} = 0, \frac{\partial \pi_{B2}}{\partial P_{B2}} = 0 \right\} \text{ yields the equilibrium prices (for either case).}$$

Substituting these prices into firm A 's profit function gives the equilibrium profits as:

$$\pi_A^{closed} = \frac{(t-2\alpha)(3t-4\alpha)(2R-2C+t)^2(7t^3+8t^2\alpha-10t\alpha^2-8\alpha^3)^2}{(t-\alpha)(35t^4-21t^3-114t^2\alpha^2+48t\alpha^3+64\alpha^4)^2}, \text{ and}$$

$$\pi_A^{open} = \frac{t(3t-4\alpha)(2R-2C+t)^2(7t^2-24\alpha^2)^2}{(t-2\alpha)(7t-6\alpha)^2(5t^2-16\alpha^2)^2}.$$

Proof of Proposition 7

$$\begin{aligned} & \pi_A^{open} - \pi_A^{closed} > 0 \\ \Leftrightarrow & \frac{t(3t-4\alpha)(2R-2C+t)^2(7t^2-24\alpha^2)^2}{(t-2\alpha)(7t-6\alpha)^2(5t^2-16\alpha^2)^2} \\ & \quad - \frac{(t-2\alpha)(3t-4\alpha)(2R-2C+t)^2(7t^3+8t^2\alpha-10t\alpha^2-8\alpha^3)^2}{(t-\alpha)(35t^4-21t^3-114t^2\alpha^2+48t\alpha^3+64\alpha^4)^2} > 0 \\ \Leftrightarrow & 73745t^{13} - 143521t^{12}\alpha - 8444137t^{11}\alpha^2 + 1705196t^{10}\alpha^3 \\ & \quad + 3701612t^9\alpha^4 - 7994848t^8\alpha^5 - 7560576t^7\alpha^6 + 18560960t^6\alpha^7 + 6667520t^5\alpha^8 \\ & \quad - 21769216t^4\alpha^9 - 1327104t^3\alpha^{10} + 11681792t^2\alpha^{11} - 393216t\alpha^{12} - 2359296\alpha^{13} > 0 \end{aligned}$$

Dividing by $\alpha^{13} > 0$, the left hand side of this inequality can be written as a 13th order polynomial in $X = \frac{t}{\alpha}$, so

$$\begin{aligned} & \pi_A^{open} - \pi_A^{closed} > 0 \\ \Leftrightarrow & 73745X^{13} - 143521X^{12} - 8444137X^{11} + 1705196X^{10} + 3701612X^9 \\ & \quad - 7994848X^8 - 7560576X^7 + 18560960X^6 + 6667520X^5 - 21769216X^4 \\ & \quad - 1327104X^3 + 11681792X^2 - 393216X - 2359296 > 0 \end{aligned}$$

Let $PN^n(X)$ denote the n 'th derivative of this polynomial, evaluated at X .

Recall that $X > 2$ by Assumption 2. It can be easily verified that

- (a) $PN^{13}(X) > 0$, and
- (b) $PN^n(2) > 0$ for all $n = 0 \dots 13$.

We use induction to show that $PN^n(X) > 0$ for $X > 2$ and all $n = 0 \dots 13$.

Clearly, (a) implies the statement for $n = 13$. If the statement is true for n , $PN^{n-1}(X)$ is increasing, so (b) ensures the induction statement is also true for $n-1$. This completes the induction and the proof of Proposition 7. ■